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Boros, George (1-NORL); **Moll, Victor H.** (1-TULN)

An integral with three parameters. (English summary)

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Article

The integral

$$I(a,b;r) := \int_0^\infty \left[\frac{x^2}{x^4+2ax^2+1}\right]^r \cdot \frac{x^2+1}{x^b+1} \cdot \frac{dx}{x^2}$$

is evaluated in closed form. The main theorem states that the following four integrals have a common value, i.e., if

$$I_1 := \int_0^\infty \left[\frac{x^2}{x^4+2ax^2+1}\right]^r \cdot \frac{x^2+1}{x^b+1} \cdot \frac{dx}{x^2},$$

$$I_2 = \int_0^\infty \left[\frac{x^2}{x^4+2ax^2+1}\right]^r \cdot \frac{dx}{x^2},$$

$$I_3 = \int_0^\infty \left[\frac{x^2}{x^4+2ax^2+1}\right]^r dx,$$

$$I_4 = \frac{1}{2}\int_0^\infty \left[\frac{x^2}{x^4+2ax^2+1}\right]^r \cdot \frac{x^2+1}{x^2}\, dx,$$

then $I_1 = I_2 = I_3 = I_4 = 2^{-(1/2)-r}(1+a)^{(1/2)-r}B(r-\frac{1}{2},\frac{1}{2})$.

The authors develop more integrals from the general integral by differentiation with respect to the parameters a and b.

There is a discussion in the paper about how two software packages evaluate, correctly or incorrectly, these integrals or their special cases. This information is useful to those who are not conversant with the power of the methods of basic analysis found in such texts as Edwards' classical treatise. The calculation times stated are dependent on the algorithms used by the programmers. Mathematica and Maple, in spite of using services of many mathematicians, have still a lot of

lacunas. The school of education espousing use of these packages may pay heed to the methods of analysis such as the one presented here. This work may revive inclusion of such methods in textbooks instead of dismissing them as drudgery fit for a computer.

Reviewed by *R. N. Kalia*