Scale free networks

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For the Lagniappe term we will be continue the seminar on Scale-free Networks. These networks arise in a remarkable number of areas: computer networks, such as the World Wide Web, social networks, the networks formed by spreading diseases, and areas of physics, as well as several other areas as well. As opposed to random networks, where one expects the number of edges each node has to be uniformly distributed, in scale free networks, the number of other nodes each node is connected to by an edge obeys a power law, and this implies that there are a few "highly connected" nodes in such a graph, while most nodes have relatively few edges. For a brief introduction and history of the area, see Wikipedia's page. A related phenomenon is the "small world phenomenon," in which adding a small number of edges randomly to a given graph results in a dramatic drop in the diameter of the graph.

We have a reading list of papers on this subject at `http://www.math.tulane.edu/~tcsem` that we would like to cover. In addition we have some research problems that we plan to explore. Here are some of the basic ideas:

- Scale free networks are graphs whose growth is governed by laws that state that, as new nodes are added, they also add edges to those nodes with the highest degree (number of edges). There are similar laws about the addition of new edges between existing nodes. It has been observed in real instances that networks like the World Wide Web grow in this manner, so their topology is that of a scale free network. We would like to investigate what would happen if additional constraints were placed on the attractiveness of existing nodes – if a node already had too many edges, then new nodes would not add edges to it because of the "congestion". What sort of model does this lead to? Can these ideas be used to predict the growth of networks like the World Wide Web in the future, as we begin to exhaust the capacity of the internet to add more edges?
- Further, we would like to derive analogs to models such as the logistic law from differential equations as potential models of networks when the resources of the network become strained. Up to now, the approach has been similar to the Malthusian law of growth, which doesn't account for the limited carrying capacity of a graph. If this is factored in, how does the growth of the network change, and when does that sort of change become apparent?
- In addition to devising abstract models for scale free networks under these sorts of "adverse" growth conditions, we would also like to derive a simulator that we could run to look at the potential

While those who have been attending the seminar this term will be familiar with the material we have already covered, the material needed to catch up is not that great. It focuses mainly on graph theory and discrete mathematics, as well as some ideas from probability theory.