MATH-11XX (DUPRÉ) FALL 2012 TEST 3 ANSWERS

DATE: WEDNESDAY 14 NOVEMBER 2012

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE T(?) QUESTIONS AND EACH IS WORTH X(?) POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

FIFTH: ALL ANSWERS MUST BE EXACT FRACTIONS OR CORRECT TO AT LEAST THREE SIGNIFICANT DIGITS.

Suppose that X is uniformly distributed with minimum value 20 and maximum value 40. Calculate:

1. $E(X) = \dfrac{20+40}{2} = 30$

2. $P(X \leq 35) = \dfrac{35-20}{40-20} = 0.75$

3. $P(X < 35) = P(X \leq 35) = 0.75$

Suppose that a box contains 20 blocks of which exactly 8 are red. We randomly draw 10 blocks from the box and count the number T of times we get a red block. Calculate:

4. $E(T) = np = (10)(8/20) = 4$

5. σ_T(given drawing with replacement)$= \sqrt{np(1-p)} = \sqrt{(4)(.6)} = \sqrt{2.4} = 1.549$

6. σ_T(given drawing without replacement)$= \left[\sqrt{np(1-p)}\right]\left[\sqrt{(N-n)/(N-1)}\right] = 1.124$

7. $P(T \leq 3|$drawing with replacement$) = 0.382$, from the binomial CDF table.

8. $P(T = 3|$drawing without replacement$) = \dfrac{C(8,3)C(12,7)}{C(20,10)}$

Suppose that we are studying the length of fish in Lake Wobegon. We have an independent random sample of 9 fish from Lake Wobegon with a sample mean length of 18 inches and a sample standard deviation of 6 inches. We assume that fish length is normally distributed for fish in Lake Wobegon.

9. What is the MARGIN OF ERROR in the 95 percent confidence interval for the true mean length of fish in Lake Wobegon if we know that the POPULATION standard deviation for fish length in Lake Wobegon is 6 inches?

$$ME = \frac{z \cdot \sigma}{\sqrt{n}} = \frac{(1.96)(6)}{\sqrt{9}} = 3.92$$

10. If we know that the POPULATION standard deviation in fish length in Lake Wobegon is 6 inches, does our sample data establish that the true mean length of the fish exceeds 14 inches at the .05 significance level? Give the value of the standardized test statistic for the sample data and give the P-Value of the data.

YES: $z_{data} = \frac{18-14}{6/\sqrt{9}} = 4/2 = 2$ and P-Value= $P(Z \geq 2) = P(Z \leq -2) = 0.0228 \leq .05$

11. What is the MARGIN OF ERROR in the 95 percent confidence interval for the true mean length of fish in Lake Wobegon if we DO NOT know that the population standard deviation for fish length in Lake Wobegon is 6 inches but instead use our sample standard deviation of 6 inches?

$$ME = \frac{t[8df] \cdot s}{\sqrt{n}} = \frac{(2.306)(6)}{\sqrt{9}} = 4.612$$

12. If we DO NOT know that the POPULATION standard deviation in fish length in Lake Wobegon is 6 inches, and instead use the sample standard deviation, does our sample establish that the true mean length of the fish exceeds 14 inches at the .05 significance level? Give the value of the standardized test statistic for the sample data and the REJECTION REGION.

YES. Rejection Region: $t[8df]_{data} \geq 1.860$ and $t_{data} = \frac{18-14}{6/\sqrt{9}} = 4/2 = 2$ and $2 > 1.86$

Suppose, in a SMALL SAMPLE, that we ask 10 ducks in Duckburg if they will vote for Donald for Mayor of Duckburg in the upcoming election. Suppose that only 2 say yes. In a LARGE SAMPLE we ask the same question of 1000 ducks and 200 say yes.

13. What is the P-Value of the SMALL SAMPLE data as evidence that the true percentage of ducks in Duckburg who currently say they will vote for Donald for Mayor of Duckburg is less than 30 percent?

P-Value= $P(\text{yes count} \leq 2|\text{binomial}, n = 10,\ p = .3) = 0.383$

14. What is the SIGNIFICANCE of the SMALL SAMPLE data as evidence that the true percentage of ducks in Duckburg who currently say they will vote for Donald for Mayor of Duckburg is less than 30 percent?

DATA SIGNIFICANCE =P-Value= $P(\text{yes count} \leq 2|\text{binomial}, n = 10,\ p = .3) = 0.383$

15. Using the LARGE SAMPLE data, what is the MARGIN OF ERROR in the 95 percent confidence interval for the true proportion of ducks who say they will vote for Donald for Mayor of Duckburg?

$$ME = \frac{z(1/2)}{\sqrt{n}} = \frac{(1.96)(1/2)}{\sqrt{1000}} = 0.0310$$