MATH-1110 (DUPRÉ) FALL 2017 TEST 3 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

PART I problems all use the information that follows. Suppose you know the fish in LAKE WOBEGONE have NORMALLY DISTRIBUTED length with a standard deviation of 6 centimeters(cm). You do not know the population mean length. Assume all fish caught are caught in LAKE WOBEGONE. Suppose that $0 < \alpha < 1$, **and the CRITICAL VALUE cutting off a RIGHT TAIL of AREA** α **on the STANDARD NORMAL DISTRIBUTION is exactly** $z = 2.3$, **whereas on the STUDENT t-DISTRIBUTION for DEGREES OF FREEDOM** $df = 8$ **it is** $t_8 = 2.9$, **for** $df = 9$ **it is** $t_9 = 2.8$, **and for** $df = 10$, **it is** $t_{10} = 2.76$.

5. If you are going to catch 9 fish and use the sample mean as your estimate for the true mean, what is your MARGIN OF ERROR with CONFIDENCE $C = 1-2\alpha$**?**

ANSWER: The MARGIN OF ERROR is $\frac{(2.3)(6)}{\sqrt{9}} = 4.6$.

6. If Joe only knows that length is normally distributed and does not know the population standard deviation but has a sample of 10 fish with sample standard deviation 6.3 cm, then what is Joe's MARGIN OF ERROR with CONFIDENCE $C = 1 - 2\alpha$**?**

ANSWER: The MARGIN OF ERROR is $\frac{(2.8)(6.3)}{\sqrt{10}}$.

7. If you catch 9 fish forming an independent random sample, and the sample mean length turns out to be 25 cm, then what is the value of YOUR TEST STATISTIC as evidence that the true mean length exceeds 20 cm?

ANSWER: The VALUE of the TEST STATISTIC is $\frac{25-20}{6/\sqrt{9}} = 2.5$.

8. If Joe only knows that length is normally distributed and does not know the population standard deviation but has a sample of 9 fish with mean 26 cm and standard deviation 6.3 cm, then what is Joe's TEST STATISTIC as evidence that the true mean length exceeds 20 cm?

ANSWER: The VALUE of the TEST STATISTIC is $\frac{26-20}{(6.3)/\sqrt{9}} = \frac{6}{2.1} = \frac{20}{7}$.

9. Suppose that you are trying to prove that the true mean length of fish exceeds 20 cm at SIGNIFICANCE LEVEL α. How big must your data test statistic be in order to know the significance of your data is conclusive at SIGNIFICANCE LEVEL α?

ANSWER: AT LEAST 2.3

10. Suppose that JOE only knows that fish length is NORMAL, he does not know the population standard deviation in length and he is trying to prove that the true mean length of fish exceeds 20 cm at SIGNIFICANCE LEVEL α with a sample of 9 fish. How big must Joe's data test statistic be in order to know the significance of HIS data is conclusive at SIGNIFICANCE LEVEL α?

ANSWER: AT LEAST 2.9

11. If in a new sample, Joe's data test statistic turns out to have the value -2.78 in trying to prove that the mean length of fish is less than 30 cm with a sample of size $n = 10$, should Joe think his data is conclusive?

ANSWER: Since Joe must use the Student t-distribution with 9 degrees of freedom and the critical value for the left tail of area α is -2.8, and as -2.78 is NOT less than or equal to -2.8, Joe should think his data is INCONCLUSIVE.

PART II problems use the information from PART I and in addition the fact that every fish in LAKE WOBEGONE is either RED or BLUE.

12. If 4 fish are caught and ALL are BLUE, then what is the signficance of this as evidence that the TRUE PROPORTION of REDFISH is LESS THAN FIFTY PERCENT?

ANSWER: THE SIGNIFICANCE OF THE DATA is the probability of finding data as or more contradictory of the NULL HYPOTHESIS that the true proportion $p \geq .5$. Since all four are blue, this means that NONE were RED, so having less than or equal to 0 red in the sample of 4 using $p = .5$ from the null hypothesis the probability is, using X for the number of redfish in the sample,

$$P(X \leq 0|p = .5,\ n = 4, X \ \ \textbf{binomial}\ ) = (1/2)^4 = \frac{1}{16}.$$

13. If 20 fish are caught and 8 are RED, then what is the MARGIN OF ERROR with CONFIDENCE $C = 1 - 2\alpha$ in the resulting estimate of $8/20$ as the true proportion of redfish?

ANSWER: The MARGIN OF ERROR is $\frac{(2.3)(.5)}{\sqrt{20}}$.

14. In large sampling, how big must the sample size be to have the MARGIN OF ERROR in the true proportion estimate at most .01 for CONFIDENCE LEVEL $C = 1 - 2\alpha$?

ANSWER: Must have the sample size AT LEAST $[\frac{(2.3)(.5)}{.01}]^2 = (115)^2 = 13225$

15. What is the value of the STANDARD ERROR used in testing the NULL HYPOTHESIS THAT the TRUE PROPORTION OF REDFISH is at most 30 PERCENT using a sample of size $n = 63$?

ANSWER: Testing the NULL HYPOTHESIS $p \leq .3$ we would use the ASSUMPTION that the TRUE PROPORTION is $p = .3$. This means the standard deviation of the indicator of a redfish is $\sigma = \sqrt{(.3)(.7)}$ and as the standard error for confidence intervals and hypothesis tests using the mean or proportion is $\sigma/\sqrt{n}$, it follows that the standard error here is $\sqrt{\frac{(.3)(.7)}{63}}$. The thing to keep in mind here with hypothesis testing for true proportions is that as soon as the null hypothesis gives a hypothetical value for the true proportion, it then implies the value of the true standard deviation, because for any indicator I_A, if $X = I_A$, and $p = P(A)$, then $\sigma_X = \sqrt{p(1-p)}$.