MATH-1110 (DUPRÉ) FALL 2011 TEST 1 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR SPRING 2011 MATH-1110 SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE TWENTY QUESTIONS AND EACH IS WORTH 5 POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

NOTATION:
$b^n =$ **the number** b **raised to the power** n.
$C(n,r) =$ **the number of ways to CHOOSE** r **things from a set of** n **things.**
$P(n,r) =$ **the number of ways to ARRANGE** r **things from a set of** n **things.**
$P(r,r) = r! =$ "r factorial".

Calculate the numerical values indicated.

1. $C(4,2) = \frac{4!}{2!2!} = \frac{4\cdot 3}{2} = 6$

2. $C(4,3) = C(4,1) = 4$

3. $C(5,3) = \frac{5!}{3!\cdot 2!} = \frac{5\cdot 4}{2} = 10$

4. $P(5,3) = C(5,3)\cdot 3! = (10)(6) = 60$

5. $\dfrac{P(7777,3)}{C(7777,3)} = \dfrac{C(7777,3)\cdot 3!}{C(7777,3)} = 3! = 6$

Suppose that a box contains 5 BLUE blocks, 3 RED blocks, and 2 GREEN blocks. Suppose that three blocks are drawn from the box without replacement one after another.

6. What is the probability that the first block drawn is red?

$P(\text{first red}) = \frac{3}{10} = 0.3$

7. What is the probability that the third block drawn is RED given that the first is GREEN and the second is BLUE?

$P(\text{third red}|\text{first green \& second blue}) = \frac{3}{8} = 0.3725$

8. What is the probability that the first block drawn is RED given that the second is BLUE and the third is RED?

$P(\text{first red} \mid \text{second blue \& third red}) = \frac{2}{8} = \frac{1}{4} = 0.25$

9. What is the probability that two of the drawn blocks are BLUE and one is RED?

$$P(\text{two blue \& one red}) = \frac{C(5,2) \cdot C(3,1)}{C(10,3)} = \frac{1}{4} = 0.25$$

Suppose in addition to the preceding information, that BLUE blocks are worth ONE dollar, that RED blocks are worth TEN dollars and green blocks are worth TWENTY dollars.

10. What is the total worth of the blocks in the box?

Total worth $= 5 \cdot 1 + 3 \cdot 10 + 2 \cdot 20 = 75$ dollars

11. If X is the value of the first block drawn, then what is $E(X)$?

$$E(X) = \frac{75}{10} = 7.50$$

12. If W is the value of the third block drawn, then what is $E(W)$?

$$E(W) = \frac{75}{10} = 7.50$$

13. If T is the total value of the three blocks drawn, then what is $E(T)$?

$$E(T) = 3 \cdot 7.50 = 22.50$$

Suppose that X is an unknown which has the possible values 1,2,3,5,6, with probabilities
.2, .1, .1, .3, p, repectively.

14. What is the probability that X is 6?

$$P(X = 6) = p = 1 - (.2 + .1 + .1 + .3) = 1 - .7 = 0.3$$

15. What is the probability that X is odd?

$P(X \text{ is odd}) = P(X = 1) + P(X = 3) + P(X = 5) = .2 + .1 + .3 = 0.6$

16. What is the probability that $X \leq 3$?

$$P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = .2 + .1 + .1 = 0.4$$

17. What is the expected value of X?

$$E(X) = (1) \cdot (.2) + (2) \cdot (.1) + (3) \cdot (.1) + (5) \cdot (.3) + (6) \cdot (.3) = 4$$

Suppose that we are writing a string of letters in line (that is a "word") taken from the alphabet A, B, C, D, K, L, M.

18. How many words of length 5 letters are possible if the same letter can be used more than once?

$$7^5 = 16807$$

19. How many 5 letter words are possible if all the letters in the word must be different?

$$P(7,5) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$$

20. How many 10 letter words are possible which have three A's, four B's and three K's?

$$\frac{10!}{3!4!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{3!3!} = 10 \cdot 3 \cdot 4 \cdot 7 \cdot 5 = 4200$$

Also we can notice that

$$\frac{10!}{3!4!3!} = \frac{10 \cdot 9 \cdot 8 \cdot C(7,3)}{3 \cdot 2} = 120 \cdot C(7,3) = 120 \cdot 35 = 4200.$$