MATH-1110 (DUPRÉ) FALL 2011 TEST 2 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR FALL 2011 MATH-1110 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

Suppose K is the statement that the racoons in my woods have normally distributed weight with mean weight 10 pounds with a standard deviation of 2 pounds. Suppose that a racoon henceforth referred to as "the racoon" is taken from my woods with weight X.

1. What is the optimal guess for the weight of the racoon, that is, what is $E(X|K)$?

ANSWER:

$$E(X|K) = \mu_X = 10.$$

FINAL ANSWER: 10

2. What is $P(X \leq 11.8|K)$?

ANSWER:
Since X is normal, we must use the tables for the standard normal. This means we must standardize the inequality $X \leq 11.8$. Since $\mu_X = 10$ and $\sigma_X = 2$, we have

$$Z_X = \frac{X - \mu_X}{\sigma_X} = \frac{X - 10}{2}.$$

The standardization of the inequality $X \leq x$ is therefore.

$$Z \leq z, \text{ where } z = \frac{x - 10}{2}.$$

In the case here, we have $x = 11.8$, so the standardization is $z = 1.8/2 = 0.9$, and therefore the standardization of the inequality $X \leq 11.8$ is the inequality

$$Z \leq 0.9.$$

Therefore,

$$P(X \leq 11.8|K) = P(Z \leq 0.9|K) = F_Z(0.9).$$

Thus, we must look up 0.9 in the standard normal cumulative distribution table in the textbook, TABLE 3, page 688 and 689 in the textbook. The result is

$$F_Z(0.9) = 0.8159.$$

FINAL ANSWER: 0.8159

3. What is $P(X \geq 8.2|K)$?

ANSWER:
As in the preceeding problem, we must standardize the inequality. The standardization of $x = 8.2$ is $z = (8.2 - 10)/2 = (-1.8)/2 = -0.9$, and therefore, the standardization of the inequality $X \geq 8.2$ is the inequality $Z \geq -0.9$. But,

$$P(Z \geq -0.9) = 1 - P(Z < -0.9) = 1 - P(Z \leq -0.9) = 1 - F_Z(-0.9),$$

because as Z is continuous, $P(Z = -0.9) = 0$. Therefore

$$P(X \geq 8.2) = 1 - F_Z(-0.9) = 1 - (.1841) = .8159.$$

Notice that the answer is the same as for the previous problem. In fact we could notice that from the symmetry of the standard normal curve,

$$P(Z \geq z) = P(Z \leq -z),$$

and therfore F_Z, the standard normal cumulative distribution function must satisfy

$$1 - F_Z(z) = F_Z(-z), \quad \text{for all } z.$$

So here, in our problem we see that

$$P(X \geq 8.2) = P(Z \geq -.9) = P(Z \leq .9) = .8159.$$

FINAL ANSWER: 0.8159

4. What is $P(8.2 \leq X \leq 11.8|K)$?

ANSWER:
We can use the laws of probability to see that

$$P(8.2 \leq X \leq 11.8) = P(X \leq 11.8) - P(X < 8.2),$$

but as X is continuous, $P(X = 8.2) = 0$, so $P(X < 8.2) = P(X \leq 8.2)$. We have

$$P(X \leq 8.2) = P(Z \leq -.9) = 0.1841,$$

and

$$P(X \leq 11.8) = P(Z \leq .9) = 0.8159,$$

therefore

$$P(8.2 \leq X \leq 11.8) = .8159 - .1841 = 0.6318.$$

FINAL ANSWER: 0.6318

5. What is $P(X \leq 11.8|[8.2 \leq X]\&K)$?

ANSWER:
In general, $P(A\&B) = P(A|B)P(B)$, so

$$P(A|B) = \frac{P(A\&B)}{P(B)}.$$

Applying this to our problem, we have, as statements,

$$[8.2 \leq X \leq 11.8]$$

is the same as

$$[8.2 \leq X]\&[X \leq 11.8],$$

so

$$P(X \leq 11.8|\ [8.2 \leq X]) = \frac{P(8.2 \leq X \leq 11.8)}{P(8.2 \leq X)} = \frac{P(8.2 \leq X \leq 11.8)}{P(X \geq 8.2)}$$
$$= \frac{.6318}{.8159} = 0.7743596029 \quad \text{or} \quad 0.774.$$

FINAL ANSWER: 0.774

Suppose that 60 percent of the ducks in Duckburg say they will vote for Goofy for mayor, and 20 ducks are asked in a survey whether or not they will vote for Goofy for mayor of Duckburg.

6. What is the probability that the number of ducks in the survey who say they will vote for Goofy for mayor of Duckburg is less than 14 but more than 9?

ANSWER:
Letting T denote the number of ducks in the survey who say they will vote for Goofy for mayor, we know that for all practical purposes T is binomially distributed with success rate $p = 0.6$ and number of independent trials equal to the sample size $n = 20$. Using F_T to denote the cumulatve distribution function for T, the values of F_T are tabulated in tables in the textbook.

$$P(T \leq k) = F_T(k), \quad \text{for any} \quad k \quad \text{with } 0 \leq k \leq 20.$$

We must keep in mind that T is discrete (as it is a count), so that

$$P(T < k) = P(T \leq k-1) = F_T(k-1).$$

Therefore

$$P(9 < T < 14) = P(T \leq 13) - P(T \leq 9) = F_T(13) - F_T(9)$$

and from the table on page 684 of the textbook we see that $F_T(13) = .750$ and $F_T(9) = .128$, so

$$P(9 < T < 14) = .750 - .128 = 0.622.$$

FINAL ANSWER: 0.622

Suppose that a box contains 5 RED blocks and 15 BLUE blocks. Suppose that 8 blocks are drawn from the box one after another.

7. What is the probability that the second block drawn is red?

ANSWER:
We can imagine that the blocks are stacked and that we know nothing of where the blocks are in the stack, we just know there are 5 red and 15 blue, and that the blocks are taken from the top of the stack. Therefore, the second block drawn will be the second block from the top of the stack, that is the block just below the top block. Since it is as likely to be any of the 20 blocks, the chance it is a red block is simply $5/20 = 1/4 = 0.25$.

FINAL ANSWER: 1/4=0.25

8. What is the expected number of BLUE blocks drawn?

ANSWER:
Since the proportion of blue blocks in the box is $3/4$, it follows that when drawing 8 blocks we should expect to get $(3/4) \cdot 8 = 6$ blue blocks.

FINAL ANSWER: 6

9. What is the VARIANCE in the number of BLUE blocks drawn, if the blocks are drawn WITH REPLACEMENT?

ANSWER:
When drawing with replacement, the succesive draws are independent which means independent random sampling (IRS), so the number of blue blocks is binomially distributed with success rate $p = 3/4$ and number of trials $n = 8$. The variance is therefore

$$\sigma^2_{IRS} = np(1-p) = 8 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{2} = 1.5.$$

FINAL ANSWER: 3/2 or 1.5

10. What is the VARIANCE in the number of BLUE blocks drawn, if the blocks are drawn WITHOUT REPLACEMENT?

ANSWER:
If the blocks are drawn without replacement, then the successive draws are no longer independent, and we are actually doing simple random sampling (SRS). This means that the count of blue blocks obeys the hypergeometric distribution, and the variance must be corrected from the result for IRS by multiplying by the SRS correction factor

$$\text{SRS correction factor } = \frac{N-n}{N-1} = \frac{20-8}{20-1} = \frac{12}{19} = 0.631578974,$$

so the variance is now

$$\sigma^2_{SRS} = \sigma^2_{IRS} \cdot \frac{12}{19} = \frac{3}{2} \cdot \frac{12}{19} = \frac{18}{19} = 0.9473684211 \text{ or about } 0.947,$$

to three significant digits.

FINAL ANSWER: 18/19 or 0.947

Suppose that X is an unknown which can ONLY have the possible values 1,2,3,5, and

$$P(X = 1) = .2,\ P(X = 2) = .3,\ P(X = 3) = .1.$$

11. What is the probability that $X = 5$?

ANSWER:
We know that by the laws of probability, the sum of the probabilities of all the possible values must add up to 1. The probabilities given only add up to 0.6, so it must be that $P(X = 5) = 1 - 0.6 = 0.4$.

FINAL ANSWER: 0.4

12. What is the expected value of X?

ANSWER:
To find the expected value of X from the distribution information, that is when we know all possible values and the probability of each value, we simply sum up the products formed by multiplying each value by its probability. Therefore

$$E(X) = (1)(0.2) + (2)(0.3) + (3)(0.1) + (5)(0.4) = .2 + .6 + .3 + 2.0 = 3.1.$$

FINAL ANSWER: 3.1

13. What is the standard deviation of X?

ANSWER:
To calculate the standard deviation we begin by calculating the variance as the standard deviation is the square root of the variance. In general, for any unknown, by definition,

$$\sigma^2_X = Var(X) = E([X - \mu_X]^2).$$

In general, for calculating by hand, this is very inefficient, but fortunately, in general, the laws of expectation give

$$\sigma^2_X = Var(X) = E(X^2) - [E(X)]^2,$$

so we could find $E(X^2)$ by using the same probabilities with the squared values. We can also note that the laws of expectation also give

$$Var(X) = Var(X-3) \text{ and } E(X-3) = 3.1 - 3 = 0.1,$$

so we can just use $X-3$ in our variance calculations instead. Thus,

$$E([X-3]^2) = (-2)^2(0.2) + (-1)^2(0.3) + (0)^2(0.1) + (2)^2(0.4) = .8 + .3 + 0 + 1.6 = 2.7,$$

and therefore the variance is

$$Var(X) = Var(X-3) = 2.5 - (.1)^2 = 2.5 - .01 = 2.69.$$

Finally, to get the standard deviation, we must take the square root of this result,

$$\sigma_X = \sqrt{2.69} = 1.640121947 \text{ or about } 1.640.$$

FINAL ANSWER: 1.640

Suppose that X is a random variable with mean μ_X and standard deviation σ_X, where

$$\mu_X = 70, \textbf{ and } \sigma_X = 10.$$

14. If $\bar{X}$ is the AVERAGE of 25 observations of X, then what is the expected value of $\bar{X}$?

ANSWER:
The expected sample average is always the true mean of the random variable sampled,

$$E(\bar{X}) = E(X) = \mu_X = 70.$$

FINAL ANSWER: 70

15. If $\bar{X}$ is the AVERAGE of 25 INDEPENDENT observations of X, then what is the standard deviation of $\bar{X}$?

ANSWER:
The average of 25 independent obervations of X is the sample mean of an independent random sample (IRS) of size $n = 25$, and we know that for IRS, the standard deviation of the sample mean is given by

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}.$$

Since the square root of 25 is 5, this means that

$$\sigma_{\bar{X}} = \frac{10}{5} = 2.$$

FINAL ANSWER: 2