

MATH-1110 (DUPRÉ) FALL 2015 LECTURE QUIZ 2 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.
2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.
3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.
4. WRITE YOUR FALL 2015 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

The remaining problems all use the information that follows. Suppose that Sam has an aquarium tank and he knows that the fish in his tank have an average weight of 12 grams with a standard deviation of 2 grams. Suppose that Sam also knows the average length of the fish in his aquarium tank is 24 mm with a standard deviation of 4 mm. Let the unknown W be the weight of a fish Joe has selected from the aquarium, which Sam does not know, and henceforth called "the fish". Let the unknown X be the length of the fish. Sam knows that the correlation of X and W is $\rho = .6$.

5. If the fish is 20 mm in length, what is the length z-score for the fish?

ANSWER: since the mean length is 24 with standard deviation 4, a length of 20 gives a z-score of $(20 - 24)/4 = -1$.

6. If Joe tells Sam the fish has length z-score 3, then what should Sam think the raw length score is for the fish?

answer: since the mean length is 24 with standard deviation 4, a z-score of 3 gives a raw score of $24 + 4(3) = 36$.

7. What should Sam predict for the fish's weight z-score using linear regression given the length z-score is 3?

ANSWER: using linear regression Sam should predict ρz for the weight z-score given z is the length z-score, which means in this case he should predict the weight z-score is $(3)(.6) = 1.8$.

8. What should Sam predict for the fish's length z-score using linear regression given the weight z-score is 2?

ANSWER: using linear regression Sam should predict ρz for the length z-score given z is the weight z-score, which means in this case he should predict the length z-score is $(2)(.6) = 1.2$.

9. What should Sam expect for his squared error when using linear regression to predict weight from length for the fish?

ANSWER: when using linear regression to predict weight from length you reduce your expected squared error by the fraction ρ^2 . Since your guess of weight without using length would cause an expected squared error $\sigma_W^2 = 2^2 = 4$, using linear regression to use length to guess weight results in an expected squared error of

$$\sigma_W^2 - \rho^2 \sigma_W^2 = (1 - \rho^2) \sigma_W^2 = (1 - .36)(4) = (.64)(4) = 2.56.$$

10. What is the covariance of X and W for Sam?

ANSWER: The covariance of X and W is $Cov(X, W)$, given by

$$Cov(X, W) = \rho \sigma_X \sigma_Y = (.6)(4)(2) = 4.8.$$

In addition, once we have the covariance, we can easily calculate the variance of $X \pm W$ from the formula

$$Var(X \pm W) = Var(X) + Var(W) \pm 2Cov(X, W).$$

For instance, the variance of $X - W$ is therefore $4^2 + 2^2 - (2)(4.8) = 20 - 9.6 = 10.4$.