MATH-1110 (DUPRÉ) FALL 2015 LECTURE QUIZ 3 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR FALL 2015 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

GIVE the probability that:

5. among ten cards chosen from a standard deck of cards there will be 4 hearts and 6 spades.

ANSWER: $C(13,4)C(13,6)/C(52,10)$.

6. among ten cards chosen from a standard deck of cards there will be 2 hearts, and 3 spades, and 5 diamonds.

ANSWER: $C(13,2)C(13,3)C(13,5)/C(52,10)$.

7. Joe has to wait more than twenty minutes for a bus at a bus stop where buses arrive on average at the rate of 6 per hour.

ANSWER: having to wait more than twenty minutes for a bus is the same as having zero buses arrive in the first twenty minutes. But twenty minutes is one third of an hour so in that time you expect 2 buses to arrive. This means the probability having to wait more than twenty minutes is the same as the Poisson probability of zero successes when you expect 2 or simply e^{-2}**. The tip off that the success count is the Poisson distribution is that the sample size is the amount of time you observe and count buses arriving, which is a continuous measure for the sample size. With the binomial and hypergeometric distribution, the sample size is a whole number of trials. You count the sample size as well as the number of successes with the binomial or hypergeometric distributions.**

8. Joe sees exactly 3 of ten cars observed to be speeding along a road where forty percent of the cars speed.

ANSWERS: here Joe has to count the cars he observes as well as the ones that are speeding, so the success count has either the binomial or hypergeometric distribution. But, the successive cars are all independent of each other or alternately, no matter how many cars he sees are speeding, the probability the next car is speeding is still forty percent. This means the success count has the binomial distribution. If X **is the number of cars speeding, then using the binomial distribution,**

$$P(X = 3|n = 10, p = .4) = C(10,3)(.4)^3(.6)^7.$$

9. Joe sees exactly 3 cars speeding while watching for an hour along a road where on average 4 speeding cars pass by per hour.

ANSWER: now the sample size, the amount observed is measured in time, a continuous measure. You need a clock or watch to measure the time. This means the success count for seeing speeding cars is governed by the Poisson distribution. If X is the number of cars found speeding during one hour where you expect 4, using the Poisson distribution,

$$P(X = 3|\mu = 4) = \frac{\mu^3 e^{-\mu}}{3!} = \frac{4^3 e^{-4}}{3!}.$$

10. the letters of the word MISSISSIPPI randomly arranged in a row would end up in alphabetical order from left to right.

ANSWER: if N is the number of ways to arrange the letters of the word MISSISSIPPI, then alphabetical order is simply one of those, so the probability of a random arrangement turning out to be alphabetical is simply $1/N$. If all the 11 letters were tagged to make them distinguishable, there would be $11!$ ways to arrange them, but since some of the letters are the same, we can reason that if we make an arrangement of the untagged letters, which can be done in N ways, and then tag the four I's which can be done in $4!$ ways, then tag the four S's which can also be done in $4!$ ways, and then tags the two P's which can be done in $2!$ ways, then by the multiplication rule we can do all these steps in $N \cdot (4!)(4!)(2!)$ ways. The end result is to end up with an arrangement of the tagged letters, but that can be done in $11!$ ways, so it must be the case that

$$N \cdot (4!)(4!)(2!) = 11!, \textbf{ so, } N = \frac{11!}{(4!)(4!)(2!)},$$

and therefore

$$P(\textbf{alphabetical}) = \frac{1}{N} = \frac{(4!)(4!)(2!)}{11!}.$$