MATH-1110 (DUPRÉ) FALL 2015 LECTURE QUIZ 4 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR FALL 2015 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

For the following problems suppose that X is the weight in pounds of a fish in my pond, and that X is normally distributed with standard deviation 5. The word fish here always means a fish from my pond.

5. If we assume the true mean of X is $\mu_X = 12$, then what is the fish weight in pounds that divides the lower 95 percent from the upper 5 percent in fish weight?

ANSWER: on the standard normal scale, a z-score of 1.645 cuts off a right tail of area .05, as we see from the table of critical values for the t-distribution, where the normal distribution is the t-distribution for infinity degrees of freedom. This means the fish weight we are looking for is simply the destandardization of 1.645. Since $\mu = 12$, by assumption, and as $\sigma = 5$, the weight which divides the lower 95 percent from the upper 5 percent is simply

$$w = 12 + (5)(1.645).$$

6. What is the chance a fish's standard score on the weight scale is between 1.23 and 1.84?

ANSWER: since the fish weight is normally distributed, the standard fish weight is simply the standard normal, so we just use the standard normal cdf table. If $F_Z(z) = P(Z \leq z)$ is the cdf function for Z, then

$$P(1.23 < Z < 1.84) = F_Z(1.84) - F_Z(1.23) = .9671 - .8907.$$

7. What is the MARGIN OF ERROR in the 95 percent CONFIDENCE INTERVAL for estimating the true mean weight μ_X from the weight of a single fish?

ANSWER: in general, for an independent random sample of size n, where the sampled variable has known standard deviation σ, the margin of error in a confidence interval, M is given by

$$M = \frac{z\sigma}{\sqrt{n}}.$$

Since we are only observing a single fish, we have $n = 1$, so the margin of error here is simply $M = z(5)$. Now, the value of z used here is determined by the specified level of confidence. On the standard scale, the middle 95 percent leaves two tails on either side having area .025 each. This means the z we are looking for is the critical value from the table of critical values which cuts off the right tail of area .025, and that is 1.960. Keep in mind we are just using the t-distribution table of critical values with infinity degrees of freedom as that is the standard normal. This means that the margin of error is M where

$$M = (1.960)(5) = 9.85.$$

8. What is the MARGIN OF ERROR in the 99 percent CONFIDENCE INTERVAL for estimating the true mean weight μ_X using the sample mean of a sample of 100 fish?

ANSWER: this is similar to the previous problem, except now the sample size n is $n = 100$, and we are increasing the level of confidence to 99 percent. To put 99 percent of the area in the middle of the standard distribution is to leave out two tails each having area .005. If we look up the critical value which cuts off the right tail of area .005 in the t-distribution table of critical values, we see that $z = 2.576$. Since $\sqrt{n} = \sqrt{100} = 10$, our margin of error is now

$$M = \frac{z\sigma}{\sqrt{n}} = \frac{(2.576)(5)}{\sqrt{100}} = \frac{(2.576)(5)}{10} = (2.576)(.5) = 1.288$$

Notice that the margin of error is now only 1.288 even though we upped the level of confidence to 99 percent whereas in the previous problem we only have 95 percent confidence and yet the margin of error is almost 10. This is the effect of taking the large sample of size $n = 100$.

9. What is the critical value, z, of the standard normal Z with the property that

$$P(Z > z) = .99?$$

ANSWER: here we need to use the symmetry of the standard normal distribution. To find the critical value cutting off a right tail of area .99 is the same as finding the value z for which $P(Z \leq z) = .01$, that is to say it cuts off a left tail of area .01. But then we see this value of z is clearly negative so $-z$ is positive and by symmetry, $-z$ cuts off a right tail of area .01. From the t-table of critical values, we see that the critical value cutting off a right tail of area .01 is 2.326. Keep in mind, we have to use infinity degrees of freedom in the t-table of critical values. Thus

$$-z = 2.326, \ \textbf{so,} \ z = -2.326.$$

10. What is the critical value, z of the standard normal Z with the property that

$$P(|Z| > z) = .01?$$

ANSWER: to say that $P(|Z| > z) = .01$ is the same as saying that $P(|Z| \leq z) = .99$, which is simply saying that z is the critical value which gives the middle 99 percent leaving tails on either side of area .005 each. In particular, from the t-table of critical values, we see z must be the critical value of the standard normal cutting off a right tail of area .005, and that is $z = 2.576$. You should draw pictures for each of these last two problems in order to visualize what is going on.