MATH-1110 (DUPRÉ) FALL 2016 TEST 1

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2016 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

The remaining problems all use the information that follows. Suppose that Joe has an aquarium tank and he tells his friend Sam that the fish in his tank have an average length of 35 millimeters with a standard deviation of 4 millimeters. Suppose that Sam is also told that the average weight of the fish in Joe's aquarium tank is 23 milligrams with a standard deviation of 3 milligrams. Suppose that Joe has selected two fish from the aquarium, one after another, without letting Sam see which fish he selected, and placed the fish in a smaller holding tank for observation. Let the unknown W_1 be the weight of the first fish Joe has selected from the aquarium, which Sam does not know, and henceforth called "the first fish". Let W_2 be the weight of the second fish Joe selected and henceforth known as "the second fish". Let the unknown X_1 be the length of the first fish, and likewise, let X_2 be the length of the second fish. Suppose Sam is told that the correlation of X_1 and W_1 is $\rho = 0.9$, which is of course the same as the correlation of X_2 with W_2.

5. Without knowing the length of the first fish Joe selected, what should Sam expect to be its weight?

ANSWER: 23

6. Without knowing the length of the second fish, what should Sam expect to be its weight?

ANSWER: 23

7. Without knowing the length of either fish, what should Sam expect to be the average weight of the two fish that Joe selected from the aquarium?

ANSWER: 23

8. Without knowing the length of the second fish, what should Sam expect for his squared error in his expected weight of the second fish?

ANSWER: 9

9. If the first fish is 43 millimeters long, what is its standardized length score?

ANSWER: 2

10. If the first fish has a standardized weight score of -3, then what is its raw weight score, that is its weight in milligrams?

ANSWER: 14

11. If Sam is told that the second fish is 35 millimeters long, then what should Sam expect for the weight of the second fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWER: 23

12. If Sam is told that the first fish is 43 millimeters long, then what should Sam expect for the weight of the first fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWER: Let x be the given length of the first fish, so $x = 43$. Then its standard length score is $z_x = (x - 35)/4 = 2$, so Sam should expect the standard weight score to be simply $z_w = \rho z_x = (.9)(2) = 1.8$ Therefore Sam should expect the actual raw weight score to be $w = 23 + (3)(1.8) = 23 + 5.4 = 28.4$

13. In the previous two cases where Sam knew the lengths of the two fish, what should Sam expect is his squared error in his expected value for the weight of the fish when using SIMPLE LINEAR REGRESSION and the correlation of length and weight to determine what weight to expect by using the length?

ANSWER: Using SLR to denote the use of Simple Linear Regression, and D to denote the unknown error, we have $E(D^2|SLR) = \sigma_W^2(1 - \rho^2) = 9(1 - .81) = 9(.19) = 1.90 - .19 =$1.71.

14. What is the percentage of reduction in expected squared error in expected weight when using SIMPLE LINEAR REGRESSION and the correlation of length with weight to calculate expected weights for fish in the aquarium?

ANSWER: 81 percent

15. What is the VARIANCE of X_1?

ANSWER: 16

16. What is the VARIANCE of W_1?

ANSWER: 9

17. If $U = X_1 + W_1$, then what is the VARIANCE of U?

ANSWER: $Var(U) = Var(X_1) + Var(W_1) + 2Cov(X_1, W_1) = 16 + 9 + 2\rho\sigma_X\sigma_W$
$= 25 + 2(.9)(4)(3) = 25 + 21.6 =$**46.6.**

18. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, then what is the correlation of X_1 with X_2?

ANSWER: $\rho =$0

19. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, and if $T = X_1 + X_2$, then what is the VARIANCE of T?

ANSWER: Since X_1 and X_2 are uncorrelated and in fact are independent, $Var(T) = \sigma_T^2 = Var(X_1) + Var(X_2) = 16 + 16 =$32.

20. If Bill guesses that the first fish is 37 millimeters long, then what should Sam expect for Bill's squared error in this guess for the length?

ANSWER: If D is the error that Bill makes, since Sam expects the length of the length of the first fish to be 35 millimeters, and expects his own squared error to be 16, he should think that a guess of 37 would give a squared error expected to be $E(D^2) = (37 - 35)^2 + 16 = 4 + 16 =$20.