

MATH-1110 (DUPRÉ) FALL 2016 TEST 2

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2016 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

The remaining problems all use the information that follows. Mickey Mouse has gone to the Duckburg Casino run by the notorious Beagle Boys. He first decides to play dice and becomes the shooter at one of the dice tables. His first step is to choose a pair of dice from a bucket of dice. The bucket contains 20 dice of which 8 are fair and 12 are loaded, but all appear identical. He rolls the chosen pair and his first roll is a three and his second roll is a five which becomes his point. He must keep rolling until he either rolls another five, in which case he wins, or rolls a seven, in which case he loses.

5. What is the numerical value of $C(20, 2)$?

ANSWER. $C(20, 2) = (20)(19)/2 = (10)(19) = 190$

6. How many ways can Mickey choose a pair of dice from the bucket so that both are fair?

ANSWER. $C(8, 2) = (8)(7)/2 = 28$

7. How many ways can Mickey choose a pair of dice from the bucket so as to have one fair dice and one loaded dice?

ANSWER. $C(8, 1)C(12, 1) = (8)(12) = 96$

8. How many ways can Mickey choose a pair of dice from the bucket?

ANSWER. $C(20, 2) = 190$

9. What is the probability that the pair of dice Mickey chooses is fair?

ANSWER. $C(8, 2)/C(20, 2) = 28/190 = 14/95$

10. If a single dice is chosen from the bucket, what is the probability that it is loaded?

ANSWER. $C(12, 1)/C(20, 1) = 12/20 = .6$

11. If the pair of dice Mickey has chosen is fair, what is the probability that he rolls a three?

ANSWER. $P(3) = 2/36 = 1/18$

12. If the pair of dice Mickey has chosen is fair, what is the probability that he rolls a five?

ANSWER. $P(5) = 4/36 = 1/9$

13. If the pair of dice Mickey has chosen is fair, what is the probability that he rolls a seven?

ANSWER. $P(7) = 6/36 = 1/6$

14. If the pair of dice Mickey has chosen is fair, what is the probability he will go on to win the game?

ANSWER. $P(5|5 \text{ or } 7) = 4/(4 + 6) = .4$

15. If the pair of dice Mickey has chosen are loaded so that the probability he rolls a five is .05 and the probability he rolls a seven is .2, then what is the probability he goes on to win the game?

ANSWER. $P(\text{WIN} | \text{LOADED DICE}) = (.05)/(.05 + .2) = 1/5 = .2$

16. If Mickey becomes suspicious of the dice and secretly takes 5 dice from the bucket to take to the Duckburg Police for testing, what is the probability that exactly 3 of the secretly chosen dice are loaded?

ANSWER. $P(3 \text{ of } 5 \text{ chosen}) = C(12, 3)C(8, 2)/C(20, 5) = .3973168215$

17. Suppose that Mickey's pair of dice are loaded so that each dice when rolled comes up even with probability .8. What is the probability that when he rolls the pair of dice, they both come up even?

ANSWER. Since the dice roll independently of each other, $P(\text{both even}) = (.8)(.8) = .64$

18. Suppose that Mickey's pair of dice are loaded so that each dice when rolled comes up even with probability .8. What is the probability that when he rolls the pair of dice, they both come up odd?

ANSWER. Since the dice roll independently of each other, $P(\text{both odd}) = (.2)(.2) = .04$

19. Suppose that Mickey's pair of dice are loaded so that each dice when rolled comes up even with probability .8. What is the probability that when he rolls the pair of dice, one comes up odd and the other comes up even?

ANSWER. The probability they are both of the same parity is the probability they are either both even or both odd, so the probability they are not the same parity, that is one even and one odd is

$P(\text{one even and one odd}) = 1 - [P(\text{both even}) + P(\text{both odd})] = 1 - .68 = .32.$

20. Suppose the Beagle Boys all know that each of the loaded dice in the bucket is loaded so that it comes up even with probability .8, and they are watching Mickey roll the dice. If Mickey rolls one of the dice chosen from the bucket and it comes up even, what do the Beagle Boys think is the probability it was one of the loaded dice (that is, what is the probability the dice rolled was a loaded dice given it comes up even)?

ANSWER. Let L be the statement that the dice is loaded, let F be the statement that the dice is fair, and E be the statement that it comes up even on the specified roll seen by the Beagle Boys. We need $P(L|E)$.

$P(L|E) = P(L \& E) / P(E)$. Also,

$$\begin{aligned} P(E) &= P(E \& L) + P(E \& F) = P(E|L)P(L) + P(E|F)P(F) \\ &= (.8)(12/20) + (.5)(8/20) = (.8)(.6) + (.5)(.4) = .48 + .2 = .68, \end{aligned}$$

and looking at the first term in the calculation gives $P(E \& L) = .48$, so finally, $P(L|E) = (.48) / (.68) = 48/68 = 24/34 = 12/17$. Notice that this means the dice are more likely to be loaded because without seeing them come up even, the probability they are loaded is only 12/20.

FINAL ANSWER: 12/17.