MATH-11X0 (DUPRÉ) SPRING 2017 TEST 2 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2017 COURSE AND SECTION NUMBERS UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

The PART I problems all use the information that follows. Mickey Mouse has gone to the Duckburg Casino run by the notorious Beagle Boys. He first decides to play dice and becomes the shooter at one of the dice tables. His first step is to choose a pair of dice from a bucket of dice. The bucket contains 50 dice of which 10 are fair and 40 are loaded, but all appear identical. He rolls the chosen pair and his first roll is a three and his second roll is a SIX which becomes his POINT. He must keep rolling until he either rolls another SIX, in which case he wins, or rolls a SEVEN, in which case he loses.

5. What is the numerical value of $C(10,3)$?

ANSWER: $C(10,3) = (10)(9)(8)/(3)(2)(1) = (10)(3)(4) = 120.$

FINAL ANSWER: 120.

6. How many ways can Mickey choose a pair of dice from the bucket so that both are fair?

FINAL ANSWER: C(10,2)=(5)(9)=45.

7. How many ways can Mickey choose a pair of dice from the bucket so as to have one fair dice and one loaded dice?

FINAL ANSWER: (10)(40)=400.

8. How many ways can Mickey choose a pair of dice from the bucket?

FINAL ANSWER: C(50,2)=(25)(49)=(4900)/4=1225.

9. What is the probability that the pair of dice Mickey chooses is fair?

FINAL ANSWER: C(10,2)/C(50,2)=(45)/(1225)=9/(245)=.0367346939.

10. If the pair of dice Mickey has chosen is fair, what is the probability that he rolls a SIX or a SEVEN?

FINAL ANSWER: (5/36)+(6/36)=(11)/(36)=.305555.....

11. If the pair of dice Mickey has chosen is fair, what is the probability he will go on to win the game?

ANSWER: Let F denote the statement that the dice are fair. The probability he goes on to win the game is the conditional probability that he rolls a SIX given F and that he rolls either SIX or SEVEN, which is

$$P(\textbf{rolls 6} \mid \textbf{rolls 6 or 7 and } F) = \frac{P(\textbf{rolls 6 } |F)}{P(\textbf{ rolls 6 or 7}|F)} = \frac{(5)/(36)}{(11)/(36)} = \frac{5}{11}.$$

FINAL ANSWER: (5)/(11)

12. If the pair of dice Mickey has chosen are loaded so that the probability he rolls a SIX is .1 and the probability he rolls a seven is .4, then what is the probability he goes on to win the game?

ANSWER: Let L denote the statement that the dice are loaded. Here the probability he goes on to win the game is the conditional probability that he rolls a SIX given L and that he rolls either SIX or SEVEN, which is

$$P(\textbf{rolls 6} \mid \textbf{rolls 6 or 7 and } L) = \frac{P(\textbf{rolls 6 } |L)}{P(\textbf{ rolls 6 or 7}|L)} = \frac{(.1)}{(.1+.4)} = \frac{1}{5} = .2.$$

FINAL ANSWER: (.1)/(.1+.4)=(.1)/(.5)=1/5=.2

13. If Mickey becomes suspicious of the dice and secretly takes 5 dice from the bucket to take to the Duckburg Police for testing, what is the probability that exactly 3 of the secretly chosen dice are loaded?

FINAL ANSWER: C(10,2)C(40,3)/C(50,5)=(45)(9880)/(2118760)=.209839176.

14. Suppose that Mickey's pair of dice are loaded so that each dice when rolled comes up ONE with probability .4. What is the probability that when he rolls the pair of dice, they both come up ONE (snake eyes)?

FINAL ANSWER: (.4)(.4)=.16.

15. Suppose that Mickey's pair of dice are loaded so that each dice when rolled comes up ONE with probability .4. What is the probability that when he rolls the pair of dice, exactly one of them comes up ONE?

ANSWER: If we think of the dice as being colored, one red and one blue, the probability that the exactly one comes up ONE can happen with either the red dice being ONE and the blue dice NOT ONE which has probability (.4)(.6)=.24, or it can happen with te blue dice being ONE and the the red dice NOT ONE, again with probability .24, so the total probability is .48. Alternately, we can realize this is a binomial probability with two trials and success rate $p = .4$ so by the binomial probability distribution formula the probability is

$$P(\textbf{one success}|\textbf{two independent trials with success rate=.4}) = C(2,1)(.4)^1(.6)^1$$
$$= (2)(.4)(.6) = .48.$$

FINAL ANSWER: (2)(.4)(.6)=.24+.24=.48.

The PART II problems concern a normal unknown or random variable X, a standard normal unknown Z, and for each positive integer k, an unknown T_k having the student $t-$distribution for k degrees of freedom. GIVEN: we look in our statistical tables and find

$$P(Z \geq 1.645) = P(T_4 \geq 2.132) = P(T_3 \geq 2.353) = .05$$
$$P(Z \geq 1.960) = P(T_4 \geq 2.776) = P(T_3 \geq 3.182) = .025$$
$$P(Z \geq 2.326) = P(T_4 \geq 3.747) = P(T_3 \geq 4.541) = .01$$
$$P(Z \geq 2.576) = P(T_4 \geq 4.604) = P(T_3 \geq 5.841) = .005.$$

16. If $\mu_X = 100$ and $P(X \leq 150) = .9$, then what is $P(50 \leq X \leq 150)$?

FINAL ANSWER: 1-(2)(.1)=1-.2=.8.

17. If $\mu = 100$ and $P(80 \leq X \leq 120) = .6$, then what is $P(X \geq 120)$?

FINAL ANSWER: (1-.6)/2=(.4)/(2)=.2.

18. If we do not know μ_X but do know $\sigma_X = 10$, and a single observation of X has value $x = 90$, then what is our confidence that μ_X is between $90 - 25.76$ and $90 + 25.76$?

FINAL ANSWER: .99 or 99 percent.

19. Suppose that we do not know μ_X but do know $\sigma_X = 10$. What is the Margin of Error in any 95 percent confidence interval based on an independent random sample of size $n = 100$.

ANSWERR: Using M for the margin of error, we have with 95 percent confidence that it will be at most

$$M = z_C \frac{\sigma_X}{\sqrt{n}} = (1.960)\frac{10}{\sqrt{100}} = 1.960.$$

Of course here, z_C denotes the critical value of the standard normal Z which cuts off a right tail of area .025, which from our table information above is 1.960.

FINAL ANSWER: 1.960.

20. Suppose that we know NEITHER the population mean NOR the population standard deviation for X but we do know that the average value of 4 independent observations of X had sample mean $\bar{x} = 103$ with sample standard deviation $s = 8$. What is the Margin of Error, M, in a 95 percent confidence interval based on this information?

ANSWER: Here we use s in place of σ_X which then requires us to replace Z by T_k where the degrees of freedom k is $k = n - 1$. Here we have $n = 4$, so $k = 3$, that is we must use 3 degrees of freedom which we see then replaces z_C by t_C which we see from out table information is 3.182 as that is the critical value cutting off a tail of area .025.

$$M = t_C \frac{\sigma_X}{\sqrt{n}} = (3.182)\frac{8}{\sqrt{4}} = (3.182)(4) = 12.728.$$

FINAL ANSWER: 12.728.