MATH-11X0 (DUPRÉ) SPRING 2017 TEST 1 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2017 MATH COURSE AND SECTION NUMBERS UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

PART I problems all use the information that follows. Suppose that Joe has an aquarium tank and he tells his friend Sam that the fish in his tank have an average length of 20 millimeters with a standard deviation of 2 millimeters and that the average weight of the fish in his aquarium tank is 30 milligrams with a standard deviation of 5 milligrams. Suppose that Joe has selected two fish from the aquarium, one after another, without letting Sam see which fish he selected, and placed the fish in a smaller holding tank for observation. Let the unknown W_1 be the weight of the first fish Joe has selected from the aquarium, which Sam does not know, and henceforth called "the first fish". Let W_2 be the weight of the second fish Joe selected and henceforth known as "the second fish". Let the unknown X_1 be the length of the first fish, and likewise, let X_2 be the length of the second fish. Suppose Sam is told that the correlation of X_1 and W_1 is $\rho = 0.8$, which is of course the same as the correlation of X_2 with W_2.

5. Without knowing the length of the second fish, what should Sam expect to be its weight?

FINAL ANSWER: 30 milligrams

6. Without knowing the length of either fish, what should Sam expect to be the average weight of the two fish that Joe selected from the aquarium?

FINAL ANSWER: 30 milligrams

7. Without knowing the length of the second fish, what should Sam expect for his squared error in his expected weight of the second fish?

FINAL ANSWER: 25

8. If the first fish has a standardized weight score of -2, then what is its raw weight score, that is its weight in milligrams?

FINAL ANSWER: 20 milligrams

9. If Sam is told that the second fish is 26 millimeters long, then what should Sam expect for the weight of the second fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWER: For $x = 26$, the standard length score is

$$z_x = (26 - 20)/2 = 3,$$

so using SLR Sam should expect the standard weight score to be

$$z_w = \rho z_x = (.8)(3) = 2.4.$$

A standard weight score of 2.4 gives a raw weight sccore of

$$w = 30 + (5)(2.4) = 30 + 12 = 42.$$

FINAL ANSWER: 42 milligrams

10. If Sam is told that the first fish is 20 millimeters long, then what should Sam expect for the weight of the first fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWER: In this case we see that Sam is told the first fish is exactly of average length, so SLR obviously leads Sam to expect the first fish is exacly average in weight which means simply 30 miligrams.

FINAL ANSWER: 30 milligrams

11. In the previous two cases where Sam knew the lengths of the two fish, what should Sam expect is his squared error in his expected value for the weight of the fish when using SIMPLE LINEAR REGRESSION and the correlation of length and weight to determine what weight to expect by using the length?

ANSWER: The expected squared error using SLR is reduced by the factor ρ^2 which is here

$$\rho^2 = (.8)^2 = .64,$$

so the expected squared error using SLR is here

$$\sigma_W^2(1 - \rho^2) = (25)(1 - .64) = (25)(.36) = (.25)(36) = (36/4) = 9.$$

FINAL ANSWER: 9

12. If Bill guesses that the first fish is 23 millimeters long, then what should Sam expect for Bill's squared error in this guess for the length if Sam does not know the length of the first fish?

ANSWER: Sam should expect Bill's squared error to be the squared length of the hypotenuse of the right triangle with base $|23 - \mu_X| = |23 - 20| = 3$ and height $\sigma_X = 2$ which by the Pythagorean Theorem is simply

$$3^2 + 2^2 = 9 + 4 = 13.$$

FINAL ANSWER: 13

13. What is the VARIANCE of X_1?

FINAL ANSWER: 4

14. If $U = X_1 + W_1$, then what is the VARIANCE of U?

ANSWER: In general, for any X and any Y, with correlation ρ,

$$Var(X \pm Y) = Var(X) + Var(Y) = 2Cov(X, Y) \text{ and } Cov(X, Y) = \rho\sigma_X\sigma_Y.$$

Here we therefore have $Cov(X_1, W_1) = (.8)(2)(5) = 8$ so

$$Var(U) = 4 + 25 + (2)(8) = 4 + 16 + 25 = 20 + 25 = 45.$$

FINAL ANSWER: 45

15. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, then what is the correlation of X_1 with X_2?

FINAL ANSWER: 0

16. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, and if $T = X_1 + X_2$, then what is the VARIANCE of T?

ANSWER: $Var(T) = Var(X_1) + Var(X_2 = 4 + 4 = 8$, because X_1 and X_2 are independent and therefore uncorrelated, so

$$\rho = 0.$$

FINAL ANSWER: $Var(T) = 8$

PART II problems use all the following information. Joe has a standard deck of cards and takes out the cards for a royal flush in hearts, namely the ten, Jack, Queen, King, and Ace all hearts. The face cards are the Jack, the Queen, and the King. He shuffles these five cards and chooses two of them without looking to see which two he got.

17. Give the numerical values of $C(5, 2)$ and of $P(5, 2)$.

FINAL ANSWER: $P(5, 2) = (5)(4) = 20$ and $C(5, 2) = P(5, 2)/2 = 10$.

18. What is the probability that the two cards Joe ended up choosing are the Queen and the King?

FINAL ANSWER: $1/C(5, 2) = 1/(10) = 0.1$

19. What is the probability that both cards are face cards?

FINAL ANSWER: $C(3, 2)/C(5, 2) = 3/(10) = 0.3$

20. If Joe were to turn up the remaining three cards one after another, what is the probability that the result would be Jack, Queen, King in that order?

FINAL ANSWER: $1/P(5, 3) = 1/[(5)(4)(3)] = 1/(60)$