MATH-1110 (DUPRÉ) SPRING 2018 TEST 2 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

PART I problems all use the information that follows. Suppose YOU know the fish in LAKE WOBEGONE have NORMALLY DISTRIBUTED length with a standard deviation of 6 centimeters(cm). YOU do not know the population mean length. Assume all fish caught are caught in LAKE WOBEGONE. Suppose that $0 < \alpha < 1$, **and the CRITICAL VALUE cutting off a RIGHT TAIL of AREA** α **on the**

STANDARD NORMAL DISTRIBUTION is exactly $z = 2.3$, **whereas on the**

STUDENT t-DISTRIBUTION for DEGREES OF FREEDOM $df = 2$ **it is** $t_2 = 5.0$, **for** $df = 3$ **it is** $t_3 = 3.5$, **and for** $df = 4$, **it is** $t_4 = 3.0$.

ALL CONFIDENCE INTERVALS use CONFIDENCE $C = 1 - 2\alpha$.

ANSWERS: BELOW, for all answers I will use X **for the length of a fish in LAKE WOBEGONE. We use** σ_e **to denote the standard error.**

5. If you are going to catch 4 fish and use the sample mean as your estimate for the true mean, what is YOUR MARGIN OF ERROR with CONFIDENCE $C = 1 - 2\alpha$**?**

ANSWER: Using M **for margin of error,** $M = z\sigma_e$, **where** σ_e **is the standard error which is the standard deviation of the sample mean,**

$$\sigma_e = \sigma_X/\sqrt{n}.$$

This is because you know $\sigma_X = 6$. **Since** $n = 4$, **the standard error is**

$$\sigma_e = 6/\sqrt{4} = 3.$$

The critical value of Z **is** $z = 2.3$, **again because you know** X **is normal and you know** σ_X, **so in order to have confidence** C, **the margin of error is**

$$M = (2.3)(6/\sqrt{4}) = (2.3)(3) = 6.9.$$

6. If JOE only knows that length is normally distributed and does NOT know the population standard deviation but has a sample of 4 fish with sample standard deviation 7 cm, then what is Joe's MARGIN OF ERROR with CONFIDENCE $C = 1 - 2\alpha$?

ANSWER: Since JOE does NOT know the population standard deviation $\sigma_X = 6$, he must use the standard deviation from his sample data which is 7 in its place and consequently, Joe must use the critical value of the student $t-$distribution in place of the critical value of Z in his margin of error calculation. Since Joe's sample is size $n = 4$, his $t-$distribution only has 3 degrees of freedom, so his critical value of t is $t_3 = 3.5$. So for JOE, the margin of error, M is given by

$$M = (3.5)(7/\sqrt{4}) = (3.5)^2 = 12.25.$$

7. If YOU catch 25 fish forming an independent random sample, and the sample mean length turns out to be 25 cm, then what is the value of YOUR TEST STATISTIC as evidence that the true mean length exceeds 20 cm?

ANSWER: If we use TS to denote YOUR test statistic, since you know the true population standard deviation is $\sigma_X = 6$, your test statistic is here

$$TS = \frac{\bar{x}_n - \mu_0}{\sigma_X/\sqrt{n}} = \frac{25 - 20}{6/\sqrt{25}} = \frac{25}{6}.$$

You should keep in mind that YOUR test statistic, TS, has the standard normal distribution because you standardized with the true standard error.

8. If JOE only knows that length is normally distributed and does not know the population standard deviation but has a sample of 25 fish with mean 27 cm and standard deviation 7 cm, then what is Joe's TEST STATISTIC as evidence that the true mean length exceeds 20 cm?

ANSWER: If we use TS to denote JOE's test statistic, since he does NOT know the true population standard deviation is $\sigma_X = 6$, he must replace it with the his sample standard deviation $s = 7$, so the value of JOE's test statistic is here

$$TS = \frac{\bar{x}_n - \mu_0}{s/\sqrt{n}} = \frac{25 - 20}{7/\sqrt{25}} = \frac{25}{7}.$$

You should keep in mind here, that Joe's test statistic has the student $t-$distribution for $n - 1$ degrees of freedom, where n is the sample size, which this problem does not specify as it is not needed to answer the question. This is because JOE cannot actually calculate the true standard error, he is only estimating it by using the sample standard deviation as an estimate of the true population standard deviation in his standard error calculation.

9. Suppose that YOU are trying to prove that the true mean length of fish exceeds 20 cm at SIGNIFICANCE LEVEL α. How big must YOUR data test statistic be in order to know the significance of your data is conclusive at SIGNIFICANCE LEVEL α? In other words, what is the critical value of the test statistic for this test to prove the true mean exceeds 20?

ANSWER: Since YOU know the population standard deviation, your test statistic has the standard normal distribution. This means for your data to be conclusive at level of significance α, your test statistic must be at least the critical value of the standard normal Z which cuts off a right tail of area α. Therefore the critical value your test statistic needs to be at least, is simply $z = 2.3$ in order for your data to be conclusive and prove the true mean exceeds 20.

10. Suppose that JOE only knows that fish length is NORMAL, he does not know the population standard deviation in length and he is trying to prove that the true mean length of fish exceeds 20 cm at SIGNIFICANCE LEVEL α with a sample of 5 fish. How big must Joe's data test statistic be in order to know the significance of HIS data is conclusive at SIGNIFICANCE LEVEL α?

ANSWER: Since JOE only knows the population is normal and does not know the population standard deviation $\sigma_X = 6$, he must use the standard deviation of his sample in its place which means his test statistic is NOT the normal distribution, it is the student $t-$distribution for $n-1$ degrees of freedom. Since his sample is of size $n = 5$, his $t-$distribution has 4 degrees of freedom so his test statistic must be at least $t_4 = 3.0$ in order to know his data is conclusive and therefore proving that the true mean exceeds 20 at level of significance α.

PART II problems that follow all suppose that Joe ONLY knows that YOUR favorite fish is AT LEAST 20 cm long BUT is NO MORE THAN 30 cm long.

11. What is the NAME OF THE DISTRIBUTION in fish length of YOUR favorite fish that JOE uses based on his information?

ANSWER: THE UNIFORM DISTRIBUTION. Whenever the only thing a person knows is the minimum and maximum values for an unknown, then the distribution for that person is the UNIFORM distribution.

12. What does JOE expect for the length of YOUR favorite fish based on his information?

ANSWER: For any unknown Y, having uniform distribution with minimum m and maximum M, the mean or expected value is

$$E(Y) = \mu_Y = \frac{m+M}{2}.$$

This means for JOE, he should expect the length of your favorite fish is

$$\mu = \frac{20+30}{2} = 25.$$

13. What does JOE think is the probability that YOUR favorite fish is over 25 cm long?

ANSWER: OBVIOUSLY, for any uniform distribution, the probability of being above average is simply $1/2$. **Since The average is here 25, the probability of being above that in length is simply** $1/2$.

14. What does JOE think is the probability that YOUR favorite fish is between 20 and 27 cm long?

ANSWER: For any unknown Y **having the uniform distribution with minimum** m **and maximum** M, **and for any numbers** a, b **with** $m \leq a \leq b \leq M$, **the probability that** $a < Y < b$ **is**

$$P(a < Y < b) = \frac{b-a}{M-m}.$$

Here this simply gives the probability for the length being between 20 and 27 as

$$P(20 < Y < 27) = (27-20)/(30-20) = .7$$

15. What does JOE think is the probability that YOUR favorite fish is between 15 and 27 cm long?

ANSWER: Since the probability the fish is less than 20 is zero, this probability is the same as the probability it is between 20 and 27 which is the same as for the previous problem,

$$P(15 < Y < 27) = P(20 < Y < 27) = \frac{27-20}{30-20} = .7.$$