MATH-1110 (DUPRÉ) SPRING 2019 TEST 2 ANSWERS

Put the required information in the upper right hand corner of each sheet as specified in the syllabus.

DIRECTIONS: This TEST is OPEN TEXT BOOK but NOT OPEN NOTES. SLR stands for SIMPLE LINEAR REGRESSION, IRS stands for INDEPENDENT RANDOM SAMPLE, SRS stands for SIMPLE RANDOM SAMPLE. CDF stands for Cumulative Distribution Function, PDF stands for Probability Density Function in case of a discrete unknown and stands for Probability Density Function in case of a Continuous unknown. Always, Z denotes an unkown having the Standard Normal distribution. For any unknown W, we denote by $\bar{W}_n$ the mean of an IRS of size n. Suppose that X and Y are INDEPENDENT NORMAL unknowns.

THREE DECIMAL PLACE ACCURACY REQUIRED.

[**1 and 2 and 3.**] **Suppose that** $\mu_X = 70$, **and** $\sigma_X = 5$.

[**1.**] **What is** $P(X > 62)$**?**

ANSWERS: $P(X > 62) = P(Z > -1.6) = P(Z \leq 1.6) = 0.9332$, **so**

$$P(X > 62) = 0.9332.$$

[**2.**] **What is the real number** x **with the property that** $P(X > x) = .9$**?**

ANSWERS: $P(Z > 1.282) = 0.1$ **so by symmetry,** $P(Z > -1.282) = .9$ **and therefore** $x = 70 - (5)(1.282) = 70 - 6.41$ **so**

$$x = 63.59.$$

[**3.**] **What is** $P(\bar{X}_4 \leq 75)$**?**

ANSWERS: $P(\bar{X}_4 \leq 75) = P(Z \leq 2.00) = 0.9772$, **so**

$$P(\bar{X}_4 \leq 75) = 0.9772.$$

[4 and 5.] Suppose that W has the Student $t-$distribution with 5 degrees of freedom.

[4.] What is $P(|W| \leq 3.365)$?

ANSWERS: From TABLE 4 Critical Values of t, 3.365 is the critical value cutting off a right tail of area 0.01 and by symmetry $|W| \leq 3.365$ leaves out both left and right tails each having area 0.01, so the total left out is 0.02 which means

$$P(|W| \leq 3.365) = 1 - 0.02 = 0.98.$$

[5.] What is the real number w with the property that $P(W \leq w) = 0.95$?

ANSWERS: From TABLE 4, the critical value cutting off a right tail area 0.05 is 2.015, so

$$w = 2.015.$$

[6 through 15.] Suppose μ_X unknown and x is a value of $\bar{X}_{25}$. Let M be the Margin of Error in a 95 percent Confidence Interval for μ_X.

[6.] What is M if we know $\sigma_X = 30$?

ANSWERS: The standard error here is $30/\sqrt{25} = 6$, so

$$M = (1.960)(6) = 11.76,$$

and therefore

$$M = 11.76.$$

[7.] What is M if we only know the BOUND $\sigma_X \leq 50$?

ANSWERS: Now we only know the standard error is less than or equal to $50/\sqrt{25} = 10$, so

$$M = (1.960)(10) = 19.6,$$

and therefore

$$M = 19.6.$$

[8.] What is M if we do not know σ_X or a bound on it, but the VARIANCE in our IRS of size 25 is 100?

ANSWERS: The standard error can only be estimated from the sample variance, so we must use the $t-$distirbution for the critical value instead of the standard normal. As $10/\sqrt{25} = 2$, and the critical value of t for 24 degrees of freedom (remember degrees of freedom here is $n - 1 = 25 - 1 = 24$) is from TABLE 4, $t_{.025} = 2.064$, so

$$M = (2.064)(2) = 4.128.$$

[9.] What is the Standard Error of this estimator of μ_X if $\sigma_X = 40$?

ANSWERS: $40/\sqrt{25} = 8$,

[10.] What is the estimate of the standard error of this estimator if we do not know σ_X or a bound for it, but have sample variance 400?

ANSWERS: $\sqrt{(400)/(25)} = 20/5 = 4$, **so the standard error estimate is 4.**

[11.] What is the probability that AT MOST 14 of the 25 observations in the sample are above average?

ANSWERS: Since any normal unknown has probability exactly 1/2 of being above average, the IRS of size 25 constitutes 25 independent observations of whether or not the value of X is above average, a yes/no question, so the distribution governing such probabilities is the BINOMIAL distribution with 25 trials and success rate 1/2, so the probability is the value of the CDF for the Binomial distribution with $n = 25$ and $p = 0.5$, which from TABLE 2 is 0.788.

[12.] If EXACTLY 14 of the 25 observations in the sample are above 75, what is the resulting Point Estimate of $P(X > 75)$?

ANSWERS: The point estimate is simply the sample proportion which is $\hat{p} = 14/25 = 0.56$, so 0.56.

[13.] If EXACTLY 14 of the 25 observations in the sample are above 75, what is the conservative Margin of Error in a 95 percent Confidence Interval for $P(X > 75)$?

ANSWERS: Since we only know $\sigma_I \leq 1/2$, for any indicator, I, whose expected value (success rate) is unknown, we should conservatively treat this as a situation where we only have the bound $B = 1/2$ for the standard deviation of the unknown whose mean we seek to estimate. Therefore, we would treat this as if the standard error is simply $(1/2)/\sqrt{25}$. Of course, we are here relying on the normal approximation to the binomial whcih requires that our sample data have at least 5 successes and at least 5 failures, which we clearly do as with $n = 25$, our 14 successes mean there were also 11 failures. Thus,

$$M = (1.960)(0.5/5) = (1.960)(0.1) = 0.1960,$$

and therefore,

$$M = 0.196.$$

[14 and 15.] Suppose in addition to the previous information about X, we have an IRS for Y of size 9 and we seek to estimate the mean difference $E(X - Y)$ using the difference of the two sample means.

[14.] if we know $\sigma_X = 15$ and $\sigma_Y = 12$, what is the Standard Error of the estimator of $E(X - Y)$?

ANSWERS: Using SE for standard error, as the two sample means are independent, their variances simply add, that is, with obvious notation,

$$SE^2 = SE_X^2 + SE_Y^2 = (15)^2/(25) + (12)^2/9$$

$$= (3^2)(5^2)/(5^2) + (3^2)(4^2)/(3^2) = 9 + 16 = 25,$$

so $SE^2 = 25$, and therefore SE=5.

[15.] If we know the standard deviations of both X and Y and the calculated Standard Error of the estimator of $E(X - Y)$ is 10, what is the Margin of Error in the 99 percent Confidence Interval for $E(X - Y)$?

ANSWERS: $M = (2.576)SE = (2.576)(10) = 25.76$, **so**

$$M = 25.76.$$