MATH-1110 (DUPRÉ) SPRING 2011 TEST 1 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR SPRING 2011 MATH-1110 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE FOURTEEN QUESTIONS AND EACH IS WORTH 7 POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

Suppose that the fish in my pond have mean length 7 inches with a standard deviation of 2 inches, and mean weight 3 pounds with a standard deviation of 1 pound. Suppose that the correlation between length and weight is $\rho = .8$ Suppose that a fish (henceforth to be referred to as "the fish") is taken from my pond with length X and weight Y.

1. What is the optimal guess for the length of the fish, that is, what is $E(X)$?

$$E(X) = \text{ mean length } = 7$$

2. What is $E(Y)$?

$$E(Y) = \text{ mean weight } = 3$$

3. What is the expected squared error if you guess the weight of the fish to be $E(Y)$ and do not know the value of X?

$$\text{expected squared error } = \sigma_Y^2 = 1^2 = 1$$

4. If you have the information that the fish is actually 11 inches long, then what is the optimal guess for the weight of the fish using that information?

$$3 + (.8)(1/2)(11 - 7) = 3 + 1.6 = 4.6$$

5. What is $E(Y|X = 9)$?

$$3 + (.8)(1/2)(9 - 7) = 3.8$$

6. What is the expected squared error in your guess of the weight of a fish you know to be a 9 inch fish when you guess $E(Y|X = 9)$?

$$(1 - \rho^2)\sigma_Y^2 = (1 - .8^2)1^2 = 1 - .64 = 0.36$$

Suppose that five cards are dealt without replacement face down from a standard deck of cards.

7. What is the probability that there are two hearts and three diamonds?

$$(13\ nCr\ 2)(13\ nCr\ 3)/(52\ nCr\ 5) = 0.00858$$

8. What is the probability that there are exactly two hearts?

$$(13\ nCr\ 2)(39\ nCr\ 3)/(52\ nCr\ 5) = 0.274$$

9. Suppose that five cards are instead drawn with replacement, the deck shuffled after each draw. What is the probability that at most two hearts were drawn?

$$binomcdf(5, 1/4, 2) = 0.896$$

Suppose that X is an unknown which has the possible values 1,2,5,6, and it is three times as likely to be even as odd.

10. What is the probability that X is odd?

$$1 = P(odd) + P(even) = P(odd) + 3P(odd) = 4P(odd),\ \text{so}\ P(odd) = 1/4$$

11. What is the probability that X is 5?

$$P(X = 1) = P(X = 5) = (1/2)P(odd) = 1/8$$

12. What is the the expected value of X given that X is odd?

$$E(X|odd) = (1 + 5)/2 = 3$$

13. What is the expected value of X?

$$E(X|even) = (2 + 6)/2 = 4,$$

so

$$E(X) = E(X|odd)P(odd) + E(X|even)P(even) = (3)(1/4) + (4)(3/4) = 15/4 = 3.75$$

14. What is the standard deviation of X?

$$E(X^2) = (1 + 25)/8 + (4 + 36)(3/4) = 146/8 = 292/16$$

and

$$[E(X)]^2 = 225/16,\ \text{so}\ Var(X) = [292 - 225]/16 = 67/16$$

therefore

$$\sigma_X = \sqrt{Var(X)} = \sqrt{67}/4 \text{ or about } 2.05$$