MATH-1110 (DUPRÉ) SPRING 2011 TEST 2 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR CORRECT SPRING 2011 MATH-1110 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE TWELVE QUESTIONS AND EACH IS WORTH 8 POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

FIFTH: Any failure to follow any part of any of the above directions will result in whatever additional loss of credit the grader wishes.

Suppose that the fish in my pond have mean length 7 inches with a standard deviation of 2 inches, Suppose that a fish (henceforth to be referred to as "the fish") is taken from my pond with length X inches. Finally, suppose that the length of fish in my pond is normally distributed.

ANSWER NOTE: We are told that X is normal, so all of the first 5 problems concern **normal distributions.** Moreover, we have $\mu_X = 7 = E(X)$ and $\sigma_X = 2$.

1. What is the probability that X is between 5 and 10?

$$P(5 \leq X \leq 10) = \text{normalcdf}(5, 10, 7, 2) = 0.7745375117,$$

which to three significant digits is

$$\mathbf{0.775}.$$

2. What is the probability that X does not exceed 7?

Since $\mu_X = 7$, this question simply asks what is the chance the fish is below average in length. Since the distribution is normal and we know that 50 percent of any normal population is below average, it follows, without even computing, that

$$\mathbf{P(X \leq 7) = 0.5}.$$

3. What is the probability that X is **EXACTLY** equal to 8?

The normal distribution is a continuous distribution and therefore we know that the probability of any exact value is **zero.**

$$\mathbf{P(X = 8) = 0}.$$

4. What is the value of x with the property that there is exactly a 70 percent chance that X is less than x?

Here we are tring to solve the equation

$$P(X \leq x) = 0.7$$

for x. This is what the inverse normal in the calculator distribution menu gives us.

$$x = \text{invNorm}(.7, 7, 2) = 8.04880102,$$

or to three significant digits

$$\mathbf{8.05}$$

5. What is the probability that the sample mean length of an independent random sample of size $n = 16$ will result in a sample mean between 6 and 7?

Here we are dealing with $\bar{X}_n$, where $n = 16$, and we know that as the sample is an independent random sample, $\bar{X}$ is normal with mean

$$\mu_{\bar{X}} = \mu_X = 7$$

and standard deviation

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{2}{\sqrt{16}} = \frac{2}{4} = 0.5.$$

Therefore

$$P(6 \leq \bar{X} \leq 7) = \text{normalcdf}\left(6, 7, 7, 2/\sqrt{(16)}\right) = 0.4772499385,$$

or to three significant digits,

$$\mathbf{0.477}.$$

Fly By Night Airways has accepted 550 reservations for a flight on their 500 seat jumbo jet. The reservations system works under the assumption that on average 90 percent of the people will show up for their reservations. Assume that is the case. Also assume that the events as to whether or not different people show up for their reservations are all independent.

ANSWER NOTE: Since a person either does or does not show up for their reservation, we are dealing with a simple **success count** of the number who show up for their reservation. The information tells us that we can assume that the probability that an individual shows up for their reservation is 90 percent. Fly By Night has accepted 550 reservations, so this constitutes a sample of size 550 from Fly By Night's population of customers. Thus, the count, T of those who show up is **binomially distributed** with $n = 550$ and $p = .9$.

6. What is the probability that exactly 490 people each show up for their reservation?

$$P(T = 490) = \text{binompdf}(550,\ .9,\ 490) = 0.0426050537,$$

or, to three significant digits,

$$\mathbf{0.0426}.$$

7. What is the probability that there will be enough seats on the jumbo jet for all the people who show up for their reservations?

Since the plane is a jumbo jet and has 500 seats, there will be enough seats if the count T of those who show up does not exceed 500. Therefore the probability that there are enough seats is

$$P(T \leq 500) = \text{binomcdf}(550,\ .9,\ 500) = 0.7806615039,$$

or, to three significant digits,

$$\mathbf{0.781}.$$

Suppose that my pond on average contains 20 tadpoles per gallon of pond water and that the number of tadpoles in disjoint regions of pond water are independent.

ANSWER NOTE: Since we are counting tadpoles, the unkown or random variable here is the tadpole count which we can denote by T. However, the sample size is the amount of water examined for the tadpole count, which has a continuous measure. All we are given is the average tadpole count per gallon of pond water, and the condition on independence of the tadpole count in disjoint regions of pond water. This means the distribution for the tadpole count is the **Posson distribution.**

8. What is the chance that 5 gallons of water from my pond will contain no more than 90 tadpoles?

$$P(T \leq 90) = \text{poissoncdf}(20 \cdot 5, 90) = 0.171385119,$$

or, to three significant digits,

$$\mathbf{0.171}.$$

9. What is the chance that 5 gallons from my pond will contain more than 90 but no more than 105 tadpoles?

$$P(90 < T \leq 105) = P(T \leq 105) - P(T \leq 90) = \text{poissoncdf}(20 \cdot 5, 105) - \text{poissoncdf}(20 \cdot 5, 90) = 0.54414227662,$$

or, to three significant digits,

$$\mathbf{0.541}.$$

10. If water is pumped out of my pond into a tank, taking the tadpoles with it in the process, what is the chance that more than two gallons will have to be pumped out before there are 41 tadpoles in the tank?

To say that more than two gallons will have to be pumped out before there are 41 tadpoles in the tank is the same as saying that the first two gallons contain no more than 40 tadpoles. Thus, if T is the tadpole count, then in two gallons we expect 20*2=40 tadpoles, so

$$P(T \leq 40) = \text{poissoncdf}(40, 40) = 0.5419181824,$$

or, two three significant digits,

$$\mathbf{0.542}.$$

Suppose that a wildlife biologist studying the spring weight of bears after just waking up from hibernation. He assumes that the weight is normally distributed with a standard deviation of 100 pounds.

ANSWER NOTE: Since the biologist assumes weight is normally distributed with standard deviation 100 pounds, he knows the distribution of the population is normal and the population standard deviation is 100 pounds, that is $\sigma = 100$. Therefore, we can use the ZInterval in the Test Menu to calculate the Margin of Error for any confidence interval using sample data on the bears.

11. In a sample of 10 bears, the sample mean weight is 650 pounds. Based on this information, what is the **MARGIN OF ERROR** for the 95 percent confidence interval for the true spring weight of bears just waking up from hibernation?

Using the invNorm, we have

$$ME = \text{invNorm}\left(.975, 0, 100/\sqrt{(}10)\right) = 61.97950328,$$

or, to three significant digits,

$$62.0.$$

If you use the ZInterval in the Test Menu, enter $\sigma_x : 10$, enter zero for $\bar{x}$, enter $n : 10$, and enter $C - level : .95$, then the readout is

$$(-61.98, 61.98)$$

which gives us the margin of error to 4 siginificant digits, and therfore the answer to three significant digits is again 62.0. If you enter $\bar{x} : 650$, the readout is

$$(588.02, 711.98),$$

from which we can again conclude that the Margin of Error is

$$ME = 711.98 - 650 = 61.98,$$

again giving the same result for the Margin of Error. Thus, the final answer is

$$\mathbf{62.0}.$$

12. In a sample of 10 bears, the sample mean weight is 650 pounds. Based on this information, what is the **MARGIN OF ERROR** for the 99 percent confidence interval for the true spring weight of bears just waking up from hibernation?

If you used the ZInterval from the Test Menu for the previous problem, you can ssimply overwrite the entry for the level of confidence changing .95 to .99. That is, to use the ZInterval for this problem, we simply enter $\sigma_x : 100$, enter $\bar{x} : 0$, enter $n : 10$, and enter $C - level : .99$. The readout gives

$(-81.45, 81.455)$, which tells us that to five significant digits, the Margin of Error is

$$ME = 81.455.$$

If you had entered $\bar{x} : 650$, the readout gives

$$(568.55, 731.45),$$

which tells us the Margin of Error to 4 significant digits is

$$ME = 731.45 - 650 = 81.45.$$

For the most accurate calculation, just use the inverse Normal from the distribution Menu,

$$ME = \text{invNorm}\left(.995, 0, 100/\sqrt{(}10)\right) = 81.45487461,$$

and in any case, we see that the Margin of Error to three significant digits is

$$\mathbf{81.5}.$$

FINAL COMMENT: You should notice that in all these problems, all problems in a given group used the same distribution. Notice how the statements of the setups in boldface print describe the information situation allowing you to determine the distribution of the unknowns you are dealing with. Once you reallize what distribution you are dealing with, the problem becomes much simpler, as the options available for a given distribution are fairly limited. For instance, the normal distribution requires a standard deviation, and if you know the unknown is normal but you do not have the standard deviation, then the distribution is a t-distribution where the standard deviation is replaced with the sample standard deviation from the sample data, and the number of degrees of freedom is $df = n - 1$, where as usual, n is the sample size. If you only know the minimum and maximum possible values of the unknown, then the distribution is the uniform distribution from the minimum to the maximum. If you have a number of independent trials and are counting successes, the distribution for the success count is a binomial distribution, where the sample size n is then the number of trials. If you have the expected number of successes per unit measure (length, area, volume, time,...), then the distribution of the success count is the Poisson distribution. If you are sampling without replacement from a small finite population, such as a deck of cards, then the distribution is the hypergeometric distribution. You should also keep in in mind that with any unknown X having a continuous distribution (for instance, normal or t- or uniform), then the probability of an exact value is always zero, $P(X = c) = 0$, no matter what number you use for c, and this means that $P(X < c) = P(X \leq c)$ no matter what number you use for c, if X has a continuous distribution. For counting distributions, on the other hand, such as the success count T, we have for any integer k, that $P(T < k + 1) = P(T \leq k)$, so

$$P(T = k) = P(T \leq k) - P(T \leq k - 1).$$

Of course, with any distribution continuous or not, we always have

$$P(a < X \leq b) = P(X \leq b) - P(X \leq a).$$