MATH-1110 (DUPRÉ) FALL 2010 TEST 4 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF THIS SHEET.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME.

THIRD: WRITE YOUR FALL 2010 MATH-1110 LAB DAY DIRECTLY UNDERNEATH YOU FIRST NAME.

1. How many three letter "words" can be made with an alphabet of only ten letters?

ANSWER: Step one choose any one of the ten letters in the alphabet for the first letter of the word, step two choose any one of the ten letters in the alphabet for the second letter of the word, and step three choose any one of the ten letters in the alphabet for the last letter of the three letter word. Obviously there are ten ways to perform each step, so by the Multiplication Principle there are $(10)(10)(10) = (10)^3 = 1000$ **ways to make a three letter word with a ten letter alphabet.**

FINAL ANSWER: $(10)^3 = 1000$

2. How many 6 letter "words" can be made with three A's two B's and a C?

ANSWER: Start by letting x **be the number of 6 letter words that can be made with three A's two B's and a C. Next we can imagine that we tag the three A's with the tags 1,2,3 so as to make all the A's distinguishable, and likewise we can tag the two C's with the tags 1,2 so as to make them distinguishable. We then have 6 distinguishable symbols and we know that the number of words which can be made with 6 different symbols is** $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$**. However, there is an alternate way to make these 720 words. Step one arrange the untagged letters, which can be done in** x **ways, step two, tag the three A's which can be done in 3! ways, and step three tag the two C's which can be done in 2! ways. Using this procedure shows that the number of words with the tagged symbols is also** $x(3!)(2!)$ **which therefore also has to be** $6! = 720$**. We therefore have the equation**

$$(3!)(2!)x = 6!$$

which we can easily solve for x **to find**

$$x = \frac{6!}{3! \cdot 2!} = 60.$$

As an alternate method, we can imagine that we have 6 blank spaces to be filled in with our letters. Step one choose three of the 6 spaces in which to put the three A's, which can be done in $C(6,3)$ **ways, step two choose two of the remaing three blank spaces to fill with the two B's, which can be done in** $C(3,2)$ **ways, and then finally there is only a single blank space left to put in the C, so there is only one way to do this last step. The Multiplication Principle then says we must have**

$$x = C(6,3) \cdot C(3,2) \cdot C(1,1) = \frac{6!}{3! \cdot 3!} \cdot \frac{3!}{2! \cdot 1!} \cdot 1 = \frac{6!}{3! \cdot 2!} = 60,$$

again when we cancel. Since $C(6,3) = 20$, and $C(3,2) = C(3,1) = 1$, can easily be found from Pascal's Triangle, this method gives the answer easily in this situation because the numbers are small.

FINAL ANSWER: 60

A postman has 15 letters to deliver to an apartment complex which has 5 mailboxes.

3. In how many ways can he do this if he disregards the addresses and simply puts three letters in each mailbox?

ANSWER: Let x be the number of ways. If we imagine that the postman puts each of the letters in a stack in each mailbox and count the number of ways that can be done, that would constitute a complete ordering of all 15 letters so there would be 15! ways to do that. But that can be done by first step one putting three letters in each mailbox which can be done in x ways, then step two stack the three letters in the first mailbox which can be done in 3! ways, step three stack the three letters in the second mailbox which can be done in 3! ways and so on, so we see that this procedure leads to $(3!)^5x$ ways to stack three letters in each mailbox. This means

$$(3!)^5x = 15!,$$

so

$$x = \frac{15!}{(3!)^5} = 168,168,000,$$

which is an enormous number of ways to put three letters in each mailbox. An alternate method here is to imagine step one choose three letters for the first mailbox which can be done in $C(15,3)$ ways, then step two choose three of the remaining 12 letters to go in the second mailbox which can be done in $C(12,3)$ ways, step three choose three of the remaining 9 letters to go in the third mailbox which can be done in $C(9,3)$ ways, step four choose three of the remaining 6 letters to go in the fourth mailbox which can be done in $C(6,3)$ ways, and step five choose three of the remaining three letters for the last mailbox, and this last step can obviously be done in only one way, since $C(3,3) = 1$. We therefore have by the Multiplication Principle that

$$x = C(15,3) \cdot C(12,3) \cdot C(9,3) \cdot C(6,3) \cdot C(3,3) = \frac{15!}{12! \cdot 3!} \cdot \frac{12!}{9! \cdot 3!} \cdot \frac{9!}{6! \cdot 3!} \cdot \frac{6!}{3! \cdot 3!},$$

which after cancelling we see is the same as the previous simpler expression for x.

FINAL ANSWER: 168,168,000

4. If he decides to disregard the addresses on the envelops and instead simply put 1 letter in the first mailbox, 2 in the second mailbox, 3 in the third mailbox, 4 in the fourth mailbox, and 5 in the fifth mailbox, in how many ways can he do this (we do not care about the order of the letters in each mailbox, it only matters which letters go in each mailbox)?

ANSWER: We can use the exact same methods here as in the previous problem. Again let x be the number of ways. The first method then leads directly to

$$x = \frac{15!}{1! \cdot 2! \cdot 3! \cdot 4! \cdot 5!} = 37837800.$$

The second method would give us

$$x = C(15,1) \cdot C(14,2) \cdot C(12,3) \cdot C(9,4) \cdot C(5,5).$$

Of course $C(5,5)=1$, **and then**

$$x = \frac{15!}{14!\cdot 1!}\cdot\frac{14!}{12!\cdot 2!}\cdot\frac{12!}{9!\cdot 3!}\cdot\frac{9!}{5!\cdot 4!} = \frac{15!}{1!\cdot 2!\cdot 3!\cdot 4!\cdot 5!},$$

again.

FINAL ANSWER: 37,837,800

5. In how many ways can he arrange all 15 letters in a stack?

ANSWER: This means arranging all 15 letters so there are

$$15! = 1307674368000$$

ways to do this. When you calculate this with the calculator, it gives the answer "1.307674368E12" which means to multiply by 10^{12} **or in other words move the decimal twelve places to the right. This is about the limit of the calculators ability to actually get the answer correct for a factorial. Larger factorials would just be approximations.**

FINAL ANSWER: 1,307,674,368,000

IMPORTANT NOTE: If you looked at the answers on the answer sheet I put on the lecturn in the lecture hall after the test, even though the expressions for the answers were correct, the numerical values given for problems 3 and 4 were wrong. This was because some of the factors were overlooked when I tried to multiply out the answers by hand. The answers here are the correct answers. I apologize if any of you were upset on seeing a numerical result different from what you correctly calculated.