MATH-1110 (DUPRÉ) FALL 2010 TEST 6 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF THIS SHEET.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME.

THIRD: WRITE YOUR FALL 2010 MATH-1110 LAB DAY DIRECTLY UNDERNEATH YOU FIRST NAME.

Supose that the large pond in my back yard has on average 5 tadpoles per gallon of pond water, independent of the particular gallon examined and that the numbers of tadpoles in non-overlapping regions of pond water are independent unknowns. Also assume that the distribution of tadpole length is normal with mean 5 millimeters and standard deviation 2 millimeters.

1. What is the probability that a tadpole in my pond is over 5 millimeters in length?

ANSWER: Any normal unknown has exactly a 50 percent chance of being above average, and here 5 millimeters is average, so the probability is .5

FINAL ANSWER: .5

2. If a fair coin is flipped 10 times, what is the probability that heads happens no more than 4 times?

ANSWER: Since the successive flips are independent, the head count, T_{10} is binomially distributed and as the coin is fair, it has a 50 percent chance of coming up heads on each flip. Therefore the probability of no more than 4 heads in 10 flips is

$$P(T_{10} \leq 4) = binomcdf(10, .5, 4) = .376953125 \text{ or } .377.$$

FINAL ANSWER: .377

3. If 10 tadpoles are randomly selected (SRS or IRS), what is the probability that no more than 4 are of above average length?

ANSWER: Each time a tadpole is selected, it has a 50 percent chance of being above average in length, so this is exactly the same as the previous problem. As you do not know the population size, it can be assume that it is so large that sampling with repacement or without repacement makes no difference, the trials are independent as far as calculation is concerned. Now T_{10} is the number of tadpoles found to be above average in length, and the success rate is still .5.

$$P(T_{10} \leq 4) = binomcdf(10, .5, 4) = .376953125 \text{ or } .377.$$

FINAL ANSWER: .377

4. If a single tadpole is selected, what is the probability it is between 4 and 7 millimeters in length?

ANSWER: If X denotes the length of the selected tadpole, then X is normal with $\mu_X = 5$ and $\sigma_X = 2$, so

$$P(4 \leq X \leq 7) = normalcdf(4, 7, 5, 2) = .5328072082 \text{ or } .533$$

FINAL ANSWER: .533

5. If we count the tadpoles in a gallon of pond water, what is the probability we find no more than 4?

ANSWER: Notice we are counting, but the size of the sample we examined must be measured, so this can only be a case where the count, T, is govenerned by the Poisson distribution.

$$P(T \leq 4) = poissoncdf(5, 4) = .4404932851 \text{ or } .440$$

FINAL ANSWER: .440

6. If 10 tadpoles from my pond are measured in length, what is the probability that their total length is between 42 and 57 millimeters?

ANSWER: Here the total T is the total for 10 independent measurements of length, so we have

$$E(T) = (10)E(X) = 50$$

but

$$\sigma_T = \sqrt{10} \cdot \sigma_X = 2\sqrt{10},$$

and as X is normal, so is T, and therefore

$$P(41 \leq T \leq 57) = normalcdf(42, 57, 50, 2\sqrt{(10)}\ ) = .7628574731 \text{ or } .763.$$

FINAL ANSWER: .763

7. If we dip a strainer into the pond, what is the probability that we have to strain more than 1/4 of a gallon of pond water before catching a tadpole? Assume that the tadpoles do not try to evade the strainer.

ANSWER: If W is the amount of water we need to strain before catching a tadpole, then to say $W > 1/4$ is exactly the same as saying that in the initial one-fourth of a gallon of pond water strained we found no tadpoles. Also, as we expect on average 5 tadpoles per gallon of pond water, in 1/4 gallon we expect $(1/4) \cdot 5 = 5/4$ tadpoles.

$$P(W > 1/4) = \mathit{poissonpdf}(5/4, 0) = .2865047969 \text{ or } .287$$

You can also notice here that as we are asking for the probability of 0 tadpoles, we can use either the cdf or the pdf and get the same result, since we cannot get a negative number of tadpoles.

FINAL ANSWER: .287

8. If we dip a strainer into the pond, what is the probability that we have to strain more than a gallon of water to catch 4 tadpoles? Assume the tadpoles do not try to evade the strainer.

ANSWER: If W_4 denotes the amount of water we must strain to catch 4 tadpoles, then to say $W > 1$ is to say that the initial gallon strained contained no more than 3 tadpoles, so as we expect 5 tadpoles per gallon,

$$P(W_4 > 1) = \mathit{poissoncdf}(5, 3) = .2650259153 \text{ or } .265$$

FINAL ANSWER: .265