MATH-11XX (DUPRÉ) FALL 2012 TEST 2 ANSWERS

DATE: WEDNESDAY 17 OCTOBER 2012

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE TWENTY(?) QUESTIONS AND EACH IS WORTH X POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

FIFTH: ALL ANSWERS MUST BE EXACT FRACTIONS OR CORRECT TO AT LEAST THREE SIGNIFICANT DIGITS.

Suppose that X and Y are unknowns with $\mu_X = 10,\ \sigma_X = 2,\ \mu_Y = 15,$ and $\sigma_Y = 3.$ Further suppose that the correlation coefficient giving the correlation of X with Y is $\rho = .8.$ Calculate the numerical values indicated for problems 1-9, using this information.

1. The VARIANCE of X is equal to

ANSWER: $Var(X) = \sigma_X^2 = 2^2 = 4$

2. If a particular score for X is $x = 13$, then the equivalent standard score is $z_x =$

ANSWER: $z_x = \frac{x-\mu_X}{\sigma_X} = \frac{13-10}{2} = 3/2 = 1.5$

3. If a particular standard score for Y is $z_y = 21$, then the actual equivalent raw score for Y is $y =$

ANSWER: $y = \mu_Y + \sigma_Y \cdot z_y = 15 + 3 \cdot 21 = 15 + 63 = 78$

4. Given that a particular score for X is equivalent to the standard score $z_x = 3$, then, using the correlation of X with Y, the corresponding standard score that should be guessed for Y is the standard score $z_y =$

ANSWER: $z_y = \rho \cdot z_x = (.8)(3) = 2.4$

5. Given a particular score $x = 11$ for X, then using the correlation of X with Y, the corresponding score that should be guessed for Y is $y =$

ANSWER: $y = \mu_Y + \frac{\rho\sigma_Y}{\sigma_X}(x - \mu_X) = 15 + (1.2)(11 - 10) = 16.2$

6. Given that we DO NOT USE the correlation of X with Y to guess a value for Y, but simply guess $y = 15$, without looking to see the value of X, then our expected squared error is equal to

ANSWER: $Var(Y) = \sigma_Y^2 = 3^2 = 9$

7. Given that we DO USE the correlation of X with Y to guess a value of Y from an observed value of X using linear regression properly, then our expected squared error is equal to

ANSWER: $(1 - \rho^2) \cdot Var(Y) = (1 - \rho^2)\sigma_Y^2 = (1 - [.8]^2)(9) = (.36)(9) = 3.24$

8. The COVARIANCE of X with Y is equal to

ANSWER: $Cov(X, Y) = \rho \cdot \sigma_X \cdot \sigma_Y = (.8)(2)(3) = (.8)(6) = 4.8$

9. The VARIANCE of $X - Y$ is equal to

ANSWER: $Var(X) + Var(Y) - 2 \cdot Cov(X, Y) = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y = 4 + 9 - 2(4.8) = 3.4$

Suppose that A and B are events with $P(A) = .4$, $P(B) = .3$, and $P(A|B) = .8$. Calculate the indicated probabilities in problems 10-14, using this information.

10. $P(A \text{ and } B) =$

ANSWER: $P(A|B)P(B) = (.8)(.3) = 0.24$

11. $P(A \text{ or } B) =$

ANSWER: $P(A) + P(B) - P(A \ \& \ B) = .4 + .3 - .24 = 0.46$

12. $P(A \text{ but not } B) =$

ANSWER: $P(A) - P(A \ \& \ B) = .4 - .24 = 0.16$

13. $P(\text{neither } A \text{ nor } B) =$

ANSWER: $1 - P(A \cup B) = 1 - .46 = 0.54$

14. $P(A \text{ or } B \text{ but not both}) =$

ANSWER: $P(A \cup B) - P(A \cap B) = .46 - .24 = 0.22$

Suppose that we have twenty cards from a standard deck of cards so as to have 5 of each suit. Suppose that we deal out three cards from this deck of twenty cards.

15. How many ways can this be done so that all three cards are of the same suit?

ANSWER: $C(4, 1) \cdot C(5, 3) = 4 \cdot 10 = 40$

16. How many ways can this be done so that there are two spades and one heart?

ANSWER: $C(5, 2) \cdot C(5, 1) = 10 \cdot 5 = 50$