MATH-1XXX (DUPRÉ) FALL 2013 TEST 3 ANSWER DETAILS

DATE: WEDNESDAY 6 NOVEMBER 2013

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

Use the attached TABLE for critical values of the t-distribution.

Suppose that the time X in minutes for a trolley to arrive is uniformly distributed with minimum value 10 and maximum value 50. Calculate:

1. $E(X) =$

A. 20

B. 25

C. 30

D. 35

E. NONE OF THE ABOVE

Remember, in general, the mean of any distribution is its balance point. For a uniform distribution, the mean is therefore the average of the minimum and maximum possible values, which in this case is obviously 30.

CORRECT ANSWER: C

2. $P(X \leq 25) =$

A. 3/8

B. 4/8

C. 5/8

D. 6/8

E. NONE OF THE ABOVE

The height, h, of a uniform distribution is easily found since the total area must be equal to 1, so obviously, if M denotes the maximum value and m denotes the minimum value, then

$$h = \frac{1}{M - m}.$$

This means that the cdf for X has the simple form

$$F_X(x) = P(X \leq x) = \frac{x - m}{M - m},\ m \leq x \leq M,$$

so in the case here,

$$P(X \leq 25) = \frac{25 - 10}{50 - 10} = \frac{15}{40} = \frac{3}{8}.$$

CORRECT ANSWER: A

3. $P(X > 25) =$

A. 3/8

B. 4/8

C. 5/8

D. 6/8

E. NONE OF THE ABOVE

Since $X > 25$ is logically equivalent to not$[X \leq 25]$, it follows that

$$P(X < 25) = 1 - P(X \leq 25) = 1 - \frac{3}{8} = \frac{5}{8}.$$

CORRECT ANSWER: C

We randomly draw 10 cards from a standard deck of cards and count the number T of times we get a spade. Calculate:

4. $E(T) =$

A. 2

B. 2.5

C. 3

D. 3.5

E. NONE OF THE ABOVE

The expected value for the total number of successes T in n trials with success rate p is always

$$E(T) = np,$$

and here $p = 0.25$ and $n = 10$, so

$$E(T) = np = (10)(0.25) = 2.5.$$

Keep in mind, that this is true whether on not we draw with or without replacement. More generally if X is any unknown which we sample, for a sample of size n, the sample total, T, is expected to be

$$E(T) = n\mu_X.$$

Here we are simply dealing with the case of an indicator, $X = I_B$, where B is the event of drawing a spade. Thus here

$$\mu_X = E(X) = E(I_B) = P(B) = p,$$

so

$$E(T) = n\mu_X = np.$$

CORRECT ANSWER: B

5. To four significant digits, σ_T(given drawing with replacement)=

A. 1.2426

B. 1.3693

C. 1.4349

D. 1.5811

E. NONE OF THE ABOVE

When drawing with replacement, the successive draws are independent of each other, so are forming an independent random sample (IRS) of the indicator I_B for drawing a spade. In general, if X is any random variable, then for an IRS if size n, the standard deviation of the sample total, T, is

$$[\sigma_T]_{IRS} = [\sqrt{n}] \cdot \sigma_X.$$

In case that $X = I_B$ is an indicator so $E(X) = P(B) = p$, we have $X^2 = X$, and therefore

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = E(X) - p^2 = p - p^2 = p(1-p).$$

This means here

$$\sigma_X = \sqrt{p(1-p} \quad \text{and} \quad \sigma_T = \sqrt{np(1-p)} = \frac{\sqrt{30}}{4} = 1.3693.$$

CORRECT ANSWER: B

6. σ_T(given drawing without replacement)=

A. 1.2426

B. 1.3693

C. 1.4349

D. 1.5811

E. NONE OF THE ABOVE

When drawing without replacement in sampling, we are forming a simple random sample (SRS), and the successive observations are no longer independent of each other. The standard deviations for the the total and sample mean must then be corrected by multiplying the IRS values by the SRS correction factor

$$\text{SRS correction factor } = \sqrt{\frac{N-n}{N-1}},$$

where N is the population size. Here, $N = 52$, and $n = 10$, so the correction factor is

$$\text{SRS correction factor } = \sqrt{\frac{42}{51}}.$$

Therefore

$$[\sigma_T]_{SRS} = \left[\sqrt{\frac{42}{51}}\right] \cdot \frac{\sqrt{30}}{4} = 1.246.$$

CORRECT ANSWER: A

7. $P(T = 3|\text{drawing with replacement}) =$

A. $C(13, 3) \cdot C(39, 7)/C(52, 10)$

B. $C(13, 3) \cdot (.25)^3 \cdot (.75)^7$

C. $C(10, 3) \cdot C(13, 7)/C(10, 7)$

D. $C(10, 3) \cdot (.25)^3 \cdot (.75)^7$

E. NONE OF THE ABOVE

When drawing with replacement, the successive draws are independent so the sample total of an indicator is then binomially distributed

$$P(T = k) = C(n, k)p^k(1 - p)^{n-k} = C(10, 3) \cdot (.25)^3 \cdot (.75)^7.$$

CORRECT ANSWER: D

8. $P(T = 3|\text{drawing without replacement}) =$

A. $C(13, 3) \cdot C(39, 7)/C(52, 10)$

B. $C(13, 3) \cdot (.25)^3 \cdot (.75)^7$

C. $C(10, 3) \cdot C(13, 7)/C(10, 7)$

D. $C(10, 3) \cdot (.25)^3 \cdot (.75)^7$

E. NONE OF THE ABOVE

When drawing without replacement and sampling an indicator, the distribution of the number of successes is hypergeometrically distributed so

$$P(T = k) = \frac{C(R, k) \cdot C(N - R, n - k)}{C(N, k)},$$

where N is the population size, and R is the number of successes in the whole population. Here we have $N = 52,\ n = 10, R = 13$, and $k = 3$, so

$$P(T = k) = \frac{C(R, k) \cdot C(N - R, n - k)}{C(N, k)} = C(13, 3) \cdot C(39, 7)/C(52, 10).$$

CORRECT ANSWER: A

Suppose that we are studying the population of bears in Smokey Mountain National Park. We have an independent random sample of 9 bears from the population with a sample mean weight of 900 pounds and a sample standard deviation of 75 pounds. We assume that bear weight is normally distributed for bears in the population.

9. What is the MARGIN OF ERROR in the 95 percent confidence interval for the true mean weight of bears in the population if we know that the POPULATION standard deviation for the weight of bears in the population is 60 pounds?

A. (1.960)(60)/3

B. (2.262)(60)/3

C. (2.262)(75)/3

D. (2.306)(75)/3

E. NONE OF THE ABOVE

Whenever the population standard deviation, σ_X, is known, for a normal population, then the sample mean random variable for independent random samples is normally distributed with mean μ_X and standard deviation $\sigma_X/\sqrt{n}$. We ignore the sample standard deviation as the population standard deviation is known. In this case we say our standard error is $\sigma_X/\sqrt{n}$. The margin of error in a confidence interval is therefore

$$ME = z \cdot \text{ standard error} = z \cdot \frac{\sigma_X}{\sqrt{n}},$$

where z denotes the critical value cutting off the standard normal distribution's upper tail of area $A = (1 - C)/2$, with C denoting the level of confidence. Here $C = .95$, so $A = .025$ and therefore $z = 1.960$. Also $n = 9$, so $\sqrt{n} = 3$, and $\sigma_X = 60$. Therefore

$$ME = z \cdot \text{ standard error} = z \cdot \frac{\sigma_X}{\sqrt{n}} = (1.960)(60)/3.$$

CORRECT ANSWER: A

10. If we know that the POPULATION standard deviation in bear weight is 60 pounds, does our sample data establish that the true mean weight of bears exceeds 850 pounds at the .01 significance level? Give the value of the standardized test statistic for the sample data and give the P-Value of the data.

A. YES, $z = 2.5$, P-Value $= P(Z > 2.5) < .01$

B. NO, $z = 2.5$, P-Value $= P(Z > 2.5) < .01$

C. YES, $t = 2$, P-Value $= P(t > 2 \mid DF = 9) < .01$

D. NO, $t = 2$, P-Value $= P(t > 2 \mid DF = 8) > .01$

E. NONE OF THE ABOVE

Whatever we attempt to establish must be the alternate hypothesis of a hypothesis test. To establish mean weight W exceeds 850 pounds means

$$H_0 : \ \mu_W = 850$$
$$vs$$
$$H_a : \mu_W > 850$$

is the hypothesis test we deal with. Since we know the population standard deviation is $\sigma_W = 60$, we ignore the sample standard deviation, and we can calculate the standardization of our sample mean, the test statistic, as

$$z = \frac{900 - 850}{(60/3)} = \frac{50}{20} = 2.5.$$

The P-Value of the data is then the probability that another sample of size $n = 9$ would give a test statistic more extreme than ours. To see what more extreme means, we simply look at the inequality in the alternate hypothesis. Therefore the P-Value of the data is

$$\text{P-Value } = P(Z > z) = P(Z > 2.5)$$

Consulting the table of critical values for infinity degrees of freedom, we see that

$$P(Z > 2.5) < .01,$$

and therefore our data does establish the alternate hypothesis at the .01 level of significance.

CORRECT ANSWER: A

11. What is the MARGIN OF ERROR in the 95 percent confidence interval for the true mean weight of bears in the population if we DO NOT know that the population standard deviation in weight of bears is 60 pounds but instead use our sample standard deviation of 75 pounds?

A. (1.960)(60)/3

B. (2.262)(60)/3

C. (2.262)(75)/3

D. (2.306)(75)/3

E. NONE OF THE ABOVE

When we do not know the population standard deviation of a normal population X, then we must estimate it using the sample standard deviation. The standardization of the sample mean is then replaced by

$$t = \frac{\bar{x} - \mu_X}{s/\sqrt{n}},$$

which has the student-t distribution for $n - 1$ degrees of freedom, and here $n - 1 = 8$. Our standard error is here really only an estimate and is

$$\text{Standard error } = \frac{s}{\sqrt{n}}.$$

This means that our margin of error ME is given by

$$ME = [t_{\text{crititcal}}] \cdot [\text{standard error}] = t_c \cdot \frac{s}{\sqrt{n}}.$$

Since we have 8 degrees of freedom here, we must look up the critical value of the student t-distribution cutting off the upper tail of area .025, which is 2.306. In the case at hand, we also have $s = 75$ and $\sqrt{n} = \sqrt{9} = 3$, so the standard error is 75/3, and therefore in this case where we do not know the population standard deviation, the margin of error is

$$ME \ = (2.306)(75)/3.$$

CORRECT ANSWER: D

12. If we DO NOT know that the POPULATION standard deviation in weight of bears is 60 pounds, and instead use the sample standard deviation, does our sample establish that the true mean weight of bears exceeds 850 pounds at the .01 significance level? Give the value of the standardized test statistic for the sample data and and give the P-Value of the data.

A. YES, $z = 2.5$, P-Value $= P(Z\ >\ 2.5)\ <\ .01$

B. NO, $z = 2.5$, P-Value $= P(Z\ >\ 2.5)\ <\ .01$

C. YES, $t = 2$, P-Value $= P(t\ >\ 2 \mid DF = 9)\ <\ .01$

D. NO, $t = 2$, P-Value $= P(t\ >\ 2 \mid DF = 8)\ >\ .01$

E. NONE OF THE ABOVE

When we do not know the population standard deviation of a normal population X, then we must estimate it using the sample standard deviation. The test statistic is standardization of the sample mean and is then estimated as

$$t = \frac{\bar{x} - \mu_X}{s/\sqrt{n}},$$

which has the student-t distribution for $n-1$ degrees of freedom, and here $n - 1 = 8$. Thus the value of the test statistic is

$$t = \frac{900 - 850}{(75/3)} = \frac{50}{25} = 2,$$

and the P-Value becomes

$$\text{P-Value } = P(t > 2|DF = 8).$$

Consulting the table of critical values for $DF = 8$, we see that the P-Value is larger than .01, so that the data does not establish the alternate hypothesis in this case where we do not know the population standard deviation.

CORRECT ANSWER: D