MATH-1XXX (DUPRÉ) FALL 2013 TEST 1 ANSWER DETAILS

DATE: WEDNESDAY 18 SEPTEMBER 2013

1. PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

2. PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

3. WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

Suppose that a dice is in a box where you cannot see it and you believe that it sits in the box with one face flat on the bottom of the box and X is the number on the top face. Calculate the numerical values indicated, based on this information and the additional information indicated, for each of the following problems.

4. The expected value of X given that the number on top is 1, 2, or 3.

A. 1

B. 2

C. 3

D. 1.5

E. None of the above

CORRECT ANSWER: B

The expected value of X is the sum over all possible values of values multiplied by their probabilities.

$$E(X|K) = \sum x \cdot P(X = x|K).$$

In particular, anytime the information does not prefer any value over any other, all must have the same probability so the expected value must then simply be the average of the possible values. Given the number up is 1,2, or 3, and no information allowing any of these three possibilities to be preferred, they must each have the same probability $= 1/3$, so the expected value is simply the average of these three values. Obviously that is 2 so the correct answer choice is **B.**

5. The expected value of X given that the number on top is 4, 5, or 6.

A. 4
B. 5
C. 6
D. 4.5
E. None of the above

CORRECT ANSWER: B

The expected value of X is the sum over all possible values of values multiplied by their probabilities.

$$E(X|K) = \sum x \cdot P(X = x|K).$$

In particular, anytime the information does not prefer any value over any other, all must have the same probability so the expected value must then simply be the average of the possible values. Given the number up is 4,5, or 6, and no information allowing any of these three possibilities to be preferred, they must each have the same probability $= 1/3$, so the expected value is simply the average of these three values. Obviously that is 5 so the correct answer choice is **B.**

6. The probability that the number on top is 1, 2, or 3.

A. 1/3
B. 1/4
C. 1/6
D. 1/2
E. None of the above

CORRECT ANSWER: D

As the possible values are all equally likely and make up the set $\{1, 2, 3, 4, 5, 6\}$, it follows that each possible value has probability $= 1/6$. Therefore, the probability that the the number on top is in the set $\{1, 2, 3\}$ is $3/6 = 1/2$, so the correct answer choice is **D.**

7. The probability that X is in the set $\{1, 2, 3\}$, given that X is 4 times as likely to be in the set $\{1, 2, 3\}$ as not.

A. 1/5
B. 1/4
C. 1/3
D. 1/2
E. None of the above

CORRECT ANSWER: E

If A is the event that the number up is in the set $\{1, 2, 3\}$, and if B is the event that the number up is not in that set, then B is simply the negation of A, so

$$P(A|K) + P(B|K) = 1,$$

whereas the information K that we are given tells us that

$$P(A|K) = 4 \cdot P(B|K).$$

It then follows that

$$1 = P(A|K) + P(B|K) = 4 \cdot P(B|K) + P(B|K) = 5 \cdot P(B|K),$$
$$\text{so } P(B|K) = 1/5 \text{ and } P(A|K) = 4/5.$$

Therefore, the correct answer choice is **E.**

8. The probability that the number on top is 2, given that X is 4 times as likely to be in the set $\{1, 2, 3\}$ as not.

A. 1/4
B. 1/(15)
C. 4/5
D. 4/(15)
E. None of the above

CORRECT ANSWER: D

If A is the event that the number up is in the set $\{1, 2, 3\}$, then as in the previous problem, we have $P(A|K) = 4/5$, and the information we are given, K, does not prefer any of these three values, so all three must be equally likely and their probabilities must add up to $4/5$, therefore

$$P(X = 1|K) = P(X = 2|K) = P(X = 3|K) = (1/3) \cdot P(A|K) = \frac{1}{3} \cdot \frac{4}{5} = \frac{4}{15}.$$

This means the correct answer choice is **D.**

9. The expected value of X given that the number on top is 4 times as likely to be in the set $\{1, 2, 3\}$ as not.

A. 4

B. 3.5

C. 2.6

D. 2..5

E. None of the above

CORRECT ANSWER: C

Let A be the event that X is in the set $\{1, 2, 3\}$ and B be the event that it is not. We then know, as in problem 7, that $P(A|K) = 4/5$ and $P(B|K) = 1/5$. We also know that as K gives no preference among the values in the set $\{1, 2, 3\}$ and likewise gives no preference among the values in the set $\{4, 5, 6\}$, then

$$E(X|A \ \& \ K) = 2 \text{ and } E(X|B \ \& \ K) = 5.$$

Therefore,

$$E(X|K) = E(X|A \ \& \ K) \cdot P(A|K) + E(X|B \ \& \ K) \cdot P(B|K) = 2 \cdot \frac{4}{5} + 5 \cdot \frac{1}{5} = \frac{13}{5} = 2.6.$$

The correct answer choice is therefore **C.**

Suppose that a box contains 2 BLUE blocks, 3 RED blocks, and 5 GREEN blocks. Suppose that three blocks are drawn from the box without replacement one after another.

10. What is the probability that the SECOND block drawn is RED?

A. 3/9

B. 3/(10)

C. 2/9

D. 2/(10)

E. None of the above

CORRECT ANSWER: B

The problem is the same information wise as if the blocks are stacked and we simply draw the blocks from the top of the stack one after another. In this view of the problem we are simply asking for the probability that the second block in the stack is red. As we have no information about any of the blocks in the stack, and as there 3 red blocks and 10 blocks in all, the probability that the second is red must be

$$P(\text{second is RED}) = \frac{3}{10}.$$

The correct answer choice is therefore **B.**

11. What is the probability that the THIRD block drawn is RED given that the FIRST is GREEN and the SECOND is BLUE?

A. 3/8
B. 3/(10)
C. 1/8
D. 2/9
E. None of the above

CORRECT ANSWER: A

Think of the blocks as being stacked and blocks are drawn from the top of the stack one after another. Now we are given the information that the block on top is GREEN and the block second from the top is BLUE, which means there are only 8 stack positions unknown to us, so the probability that the THIRD is RED given this additional information is

$$P(\text{THIRD is RED}|\text{ first is GREEN and second is BLUE}) = \frac{3}{8}.$$

The correct answer choice is therefore **A.**

12. What is the probability that the SECOND block drawn is RED given that the FIRST is BLUE and the THIRD is GREEN?

A. 3/8
B. 3/(10)
C. 1/8
D. 2/9
E. None of the above

CORRECT ANSWER: A

Think of the blocks as being stacked and blocks are drawn from the top of the stack one after another. Now we are given the information that the block on top is BLUE and the block third from the top is GREEN, which means there are only 8 stack positions unknown to us, so the probability that the second is red given this additional information is

$$P(\text{SECOND is RED}|\text{ first is BLUE and third is GREEN}) = \frac{3}{8}.$$

The correct answer choice is therefore **A.**

13. What is the probability that ALL three are GREEN?

A. $(1/2)^3$
B. (1/2)(4/9)(3/8)
C. (1/2)+(4/9)+(3/8)
D. (1/2)+(4/9)-(3/8)
E. None of the above

CORRECT ANSWER: B

For any two statements A and B, and any statement K,

$$P(A \ \& \ B|K) = P(A|B \ \& \ K) \cdot P(B|K).$$

Let G_1 be the event that the first block drawn is GREEN, let G_2 be the event that the second block drawn is GREEN, and let G_3 be the event that the third block drawn is GREEN. Then

$$\begin{aligned} P(\text{all three are GREEN}) &= P(G_1 \ \& \ G_2 \ \& \ \& G_3) = P(G_2 \ \& G_3|G_1) \cdot P(G_1) \\ &= P(G_3|G_2 \ \& \ G_1) \cdot P(G_2|G_1) \cdot P(G_1) = \frac{3}{8} \cdot \frac{4}{9} \cdot \frac{5}{10}. \end{aligned}$$

Suppose in addition to the preceding information, that GREEN blocks are worth ONE dollar, that RED blocks are worth TEN dollars and BLUE blocks are worth TWENTY dollars.

14. What is the total worth in dollars of the blocks in the box?

A. 31
B. 75
C. 150
D. 310
E. None of the above

CORRECT ANSWER: B

OBVIOUS!!

15. If W is the WORTH of the THIRD block drawn, then what is $E(W)$?

A. 3.1
B. 7.5
C. 15
D. 31
E. None of the above

CORRECT ANSWER: B

If we think of the blocks as being stacked, then without knowing any of the colors of the blocks in the stack, just that the total worth of the whole stack is 75, then we must conclude that each is expected to be worth 7.5, and in particular, as W is the worth of the third block in the stack, $E(W) = 7.5$.