MATH-1140 (DUPRÉ) FALL 2014 TEST 1 ANSWERS

Suppose that X and Y are unknowns with $\mu_X = 25$, $\sigma_X = 4$, $\mu_Y = 30$, and $\sigma_Y = 8$. Further suppose that the correlation coefficient giving the correlation of X with Y is $\rho = .8$.

1. The VARIANCE of X is equal to

ANSWER:

$Var(X) = \sigma_X^2 = 4^2 = 16$

2. If a particular score for X is $x = 31$, then the equivalent standard score is $z_x =$

ANSWER:

$z_x = (x - \mu_X)/\sigma_X = (31 - 25)/4 = 6/4 = 1.5$

3. If a particular standard score for Y is $z_y = 2.5$, then the actual equivalent raw score for Y is $y =$

ANSWER:

$y = \mu_Y + \sigma_Y \cdot z_y = 30 + 8(2.5) = 30 + 20 = 50$

4. Given a particular score $x = 31$ for X, then using the correlation of X with Y, the corresponding score that should be guessed for Y using simple linear regression (SLR) is
$y = E(Y|X = 31$ **using SLR** $) =$

ANSWER:

$y = E(Y = 31$ **using SLR** $) = \mu_Y + \rho(\sigma_Y/\sigma_X)(x - \mu_X) = \mu_Y + \rho\sigma_Y z_x = 30 + (.8)(8)(1.5) = 39.6$

5. Given that we DO NOT USE the correlation of X with Y to guess a value for Y, but simply guess $y = 30$, without looking to see the value of X, then our expected squared error is equal to

ANSWER:

$\sigma_Y^2 = 8^2 = 64$

6. Given that we DO USE the correlation of X with Y and simple linear regression to guess a value of Y from an observed value of X, then our expected squared error is equal to

ANSWER:

$(1 - \rho^2)\sigma_Y^2 = (1 - .8^2)8^2 = (.36)(64) = 23.04$

Suppose that X and Y are unknowns with $\mu_X = 25,\ \sigma_X = 4,\ \mu_Y = 30,$ and $\sigma_Y = 8$. Further suppose that the correlation coefficient giving the correlation of X with Y is $\rho = .8$.

7. The COVARIANCE of X with Y is equal to

ANSWER:

$Cov(X, Y) = \rho\sigma_X\sigma_Y = (.8)(4)(8) = 25.6$

8. The VARIANCE of $X - Y$ is equal to

ANSWER:

$Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y) = 16 + 64 - 2(25.6) = 28.8$

Suppose that A and B are events with $P(A) = .6,\ P(B) = .3,$ and $P(A|B) = .8$.

9. $P(A$ **and** $B) =$

ANSWER:

$P(A\&B) = PA|B)P(B) = (.8)(.3) = .24$

10. $P(A$ **but not** $B) =$

ANSWER:

$P(A$ **but not** $B) = P(A) - P(A\&B) = .6 - .24 = .36$

Suppose that we have a batch of 5 widgets of which two are defective. We randomly select three widgets (without replacement) and check each to see if it is defective.

11. What is the probability that two are defective?

ANSWER:

$P($ **2 defective** $) = C(2,2)C((3,1)/C(5,3) = (1)(3)/(10) = .3$

12. What is the probability that AT LEAST one is defective?

ANSWER:

$P($ **at least 1 defective** $) = 1 - P($ **0 defective** $) = 1 - C(3,3)/C(5,3) = 1 - (.1) = .9$

Suppose there are 9 fish in my pond with mean weight 7 pounds and standard deviation 2 pounds. Suppose that I plan to take a sample of $n = 5$ fish.

13. What should I expect to get for the sample total?

ANSWER:

$E(T_5) = (5)\mu = (5)(7) = 35$

14. What should I expect to get for the sample mean?

ANSWER:

$E(\bar{X}_5) = \mu = 7$

15. If I intend to use independent random sampling (IRS), then what should I expect for the VARIANCE of the sample total?

ANSWER:

$Var(T_5)_{IRS} = (5)\sigma^2 = (5)(2^2) = 20$

16. If I intend to use independent random sampling (IRS), then what should I expect for the VARIANCE of the sample mean?

ANSWER:

$Var(\bar{X}_5)_{IRS} = \sigma^2/5 = 4/5 = .8$

17. If I intend to use simple random sampling (SRS), then what should I expect for the VARIANCE of the sample total?

ANSWER:

$Var(T_5)_{SRS} = [(N-n)/(N-1)]Var(T_5)_{IRS} = [(9-5)/8](20) = (4/8)(20) = (1/2)(20) = 10$

Suppose that you are dealt three cards from a standard deck of cards. There are $C(52,3) = 22100$ possible hands that can result.

18. What is the chance you will get three aces?

ANSWER:

$P($ **three aces** $) = C(4,3)/C(52,3) = 4/22100 = 1/5525 = .0001809954751$

19. What is the chance you will get three of a kind?

ANSWER:

$P($ **three of a kind** $) = C(13,1)C(4,3)/C52,3) = 13/5525 = 1/425 = .0023529412$

20. What is the chance you will get an ace, a king, and a queen?

ANSWER:

$P($ **ace king queen** $) = [C(4,1)]^3/22100 = 4^3/22100 = 16/5525 = .0028959276$