MATH-1140 (DUPRÉ) SPRING 2016 LECTURE QUIZ 3 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2016 MATH-1110 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

5. How many 10 letter "words" can be made with the alphabet A,B,C which have exactly 2 A's, 3 B's, and 5 C's?

ANSWER: $\frac{10!}{2!\ 3!\ 5!}$ **words**

6. How many 3 card hands of cards can be made from a deck which has only 5 cards?

ANSWER: $C(5;3) = \frac{5!}{2!\ 3!}$ **hands**

The remaining problems all use the information that follows. Suppose that Sam has an aquarium tank containing ONLY 12 fish, and he knows that the fish in his tank have an average weight of 25 grams with a standard deviation of 11 grams. Joe weighs FOUR fish from Sam's aquarium, one after another, WITH REPLACEMENT (or, catch and release, in fisherman terminology) but Sam does not see which fish Joe weighs. Joe does not know anything in advance about the weights of the fish in Sam's aquarium.

7. What should Sam EXPECT is the AVERAGE weight of the FOUR fish Joe weighed?

ANSWER: 25 grams

8. What should Sam EXPECT is the squared error in his expected AVERAGE weight of the FOUR fish Joe weighed?

ANSWER: $\frac{(N-n)\sigma^2}{(N-1)n} = \frac{(12-4)(11)^2}{(12-1)4} = 22$

9. How many ways can Joe choose four fish from Sam's aquarium if he chooses WITH REPLACEMENT?

ANSWER: $(12)^4 = (144)^2 = 20736$ **ways, counting the order, but see NOTE below**

10. How many ways can Joe choose four fish from Sam's aquarium if he chooses WITHOUT REPLACEMENT?

ANSWER: $P(12;4) = \frac{12!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = (495)(24) = 12000 - 120 = 11880$ **ways, counting the order, if you do not count the order, it is only 495**

NOTE: To count the number of ways to choose 4 fish with replace but not counting the order, we cannot simply divide the $(12)^4$ by 4! because the number of rearrangements of 4 things is not simply 4! if there are repeats that is is some of the four things are alike, which can happen if we choose fish with replacement. For instance there are 12 ways that all the four fish could be the same fish, and for each one of those $12 = C(12;1)$ ways, the order already does not matter. For the case where the four fish consist of only two different fish, there are $C(12;2)$ ways to get two fish, but then there are several possibilities here, one of the fish could have been chosen only once and the other three times, for which there are 2 ways to decide which of the two fish is to be repeated 3 times, or they both could have been chosen twice, and there is only one way to do this, so there are in all $3 \cdot C(12;2)$ ways to choose 4 fish so that there are only actually two different fish. In the case that the four fish consist of 3 different fish, there are $C(12;3)$ ways to decide which three fish to choose, and then there are 3 ways to decide which of those three is to be the one to be chosen twice, so there are 3 $cdotC(12;3)$ ways to have chosen 4 fish where the four fish were actually only 3 different fish, and then for all four fish being different of course you just get $C(12;4)$. This means the total number of ways to choose four fish with replacement when order is not counted is

$$\begin{aligned} C(12;1) + 3 \cdot C(12;2) + 3 \cdot C(12;3) + C(12;4) \\ = 12 + (3)(66) + (3)(220) + 495 \\ = 12 + 198 + 660 + 495 \\ = 870 + 495 \\ = 1365 \end{aligned}$$

from which we see that if you are not paying attention to repeats when drawing with replacement, it is more likely you are getting repeats than not.