MATH-1140 (DUPRÉ) SPRING 2013 TEST 1 ANSWERS

DATE: WEDNESDAY 6 FEBRUARY 2013

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: THERE ARE TWENTY(?) QUESTIONS AND EACH IS WORTH 5 POINTS. WRITE ALL YOUR ANSWERS NEATLY IN THE SPACE PROVIDED UNDER EACH QUESTION. NEATNESS COUNTS. IF I CANNOT READ IT WITHOUT STRAINING MY EYES YOU GET NO CREDIT.

Suppose that a dice is in a box where you cannot see it and you believe that it sits in the box with one face flat on the bottom of the box and X is the number on the top face. Calculate the numerical values indicated, based on this information and the additional information indicated, for each of the following problems.

1. The expected value of X given that the number on top is 2 or 4.

ANSWER:

$$E(X|X = 2 \text{ or } 4) = 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = \frac{6}{2} = 3.$$

NOTE: From the given information, the only possible values for X are the values 2 and 4, and nothing in our information allows us to assume one is more likely than another so both are equally likely. Whenever all possible values are equally likely, the expected value is just the average of all the possible values.

2. The expected value of X given that the number on top is 4, 5, or 6.

ANSWER:

$$E(X|X \text{ is } 4,\ 5,\ \text{ or } 6) = \frac{4+5+6}{3} = \frac{15}{3} = 5$$

NOTE: Again, whenever all possible values are equally likely, the expected value is just the average of all the possible values. Nothing in the given information allows the possibility that any of the possible values is more likely than any other, so they all must be equally likely.

3. The probability that the number on top is 2 or 3.

ANSWER:

$$P(X = 2 \text{ or } 3) = P(X = 2) + P(X = 3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

NOTE: Nothing in the given information allows us to conclude that any single face of the dice is more likely to be on top then any other face, so all of the 6 faces are equally likely and therefore each face has probability $1/6$ of being the top face. Therefore

$$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6) = \frac{1}{6}.$$

4. The probability that X is even, given that X is 4 times as likely to be even as odd.

ANSWER:

$$P(X \text{ is even } | X \text{ is 4 times as likely to be even as odd }) = \frac{4}{5} = 0.8$$

NOTE: In this case we have information which tells us the even values are more likely than odd values. Let A be the statement that X is even and let B be the statement that that X is odd. Let C be the statement that X is 4 times as likely to be even as odd. Then

$$P(A|C) + P(B|C) = 1 \text{ and } P(A|C) = 4P(B|C),$$

and therefore

$$1 = P(A|C) + P(B|C) = 4P(B|C) + P(B|C) = 5P(B|C), \text{ so therefore, } P(B|C) = \frac{1}{5}.$$

But then,

$$P(A|C) = 4P(B|C) = \frac{4}{5}.$$

5. The probability that the number on top is 2, given that X is 4 times as likely to be even as odd.

ANSWER:

$$\begin{aligned} &P(X = 2 | X \text{ is 4 times as likely to be even as odd }) \\ &= \frac{1}{3} P(X \text{ is even } | X \text{ is 4 times as likely to be even as odd }) \\ &= \frac{1}{3} \cdot \frac{4}{5} = \frac{4}{15} \text{ or about } 0.267 \end{aligned}$$

NOTE: Nothing in our given information allows us to conclude that any of the even values 2,4,6 are more likely, that is they must all have the same probability given the information that X is 4 times as likely to be even as odd. Thus, if C denotes the statement that X is 4 times as likely to be even as odd, then

$$P(X = 2|C) + P(X = 4|C) + P(X = 6|C) = P(X \text{ is even } |C) = \frac{4}{5}$$

and

$$P(X = 2|C) = P(X = 4|C) = P(X = 6|C) = \frac{1}{3} P(X \text{ is even } |C = \frac{4}{15}.$$

6. The expected value of X given that the number on top is 4 times as likely to be even as odd.

ANSWER:

$$E(X|C) = E(X|(X \text{ is even })\&\ C)P(X \text{ is even } |C) + E(X|(X \text{ is odd })\&\ C)P(X \text{ is odd } |C)$$

$$= 4 \cdot \frac{4}{5} + 3 \cdot \frac{1}{5} = \frac{19}{5} = 3.8.$$

Suppose that a box contains 2 BLUE blocks, 3 RED blocks, and 5 GREEN blocks. Suppose that three blocks are drawn from the box without replacement one after another.

7. What is the probability that the SECOND block drawn is RED?

ANSWER:

$$P(\text{SECOND block drawn is RED}) = \frac{3}{10} = 0.3.$$

8. What is the probability that the THIRD block drawn is RED given that the FIRST is GREEN and the SECOND is BLUE?

ANSWER:

$$P(\text{ THIRD block drawn is RED}|\text{ FIRST is GREEN \& SECOND is BLUE }) = \frac{3}{8}.$$

9. What is the probability that the SECOND block drawn is RED given that the FIRST is BLUE and the THIRD is GREEN?

ANSWER: Same answer as the previous problem, namely 3/8.

10. What is the probability that ALL three are GREEN?

ANSWER:

$$P(\text{ ALL GREEN }) = \frac{5}{10}\frac{4}{9}\frac{3}{8} = \frac{1}{12} \text{ or about } 0.0833.$$

Suppose in addition to the preceding information, that GREEN blocks are worth EIGHT dollars, that RED blocks are worth TWENTY dollars and BLUE blocks are worth FIFTY dollars. Suppose also that W is the worth of the second block drawn and T is the total value of the three blocks drawn.

11. What is $E(W)$?

ANSWER:

$$E(W) = 8 \cdot \frac{5}{10} + (20) \cdot \frac{3}{10} + (50) \cdot \frac{2}{10} = \frac{200}{10} = 20$$

12. What is $E(T)$?

ANSWER:

$$E(T) = 3E(W) = 3 \cdot 20 = 60.$$

13. What is the VARIANCE of W?

ANSWER: To compute the variance of W which we can denote $Var(W)$, the simplest way is to use the computation formula

$$Var(W) = E(W^2) - [\mu_W]^2,$$

since from the previous problem we already know $\mu_W = 20$. Then

$$E(W^2) = (64) \cdot \frac{5}{10} + (400) \cdot \frac{3}{10} + (2500) \cdot \frac{2}{10} = \frac{320 + 1200 + 5000}{10} = \frac{6520}{10} = 652,$$

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$$Var(W) = 652 - (20)^2 = 652 - 400 = 252.$$

14. What is the STANDARD DEVIATION of W?

ANSWER: The standard deviation is always the square root of the variance, so denoting the standard deviation of W by σ_W, we have

$$\sigma_W = \sqrt{Var(W)} = \sqrt{252} = 15.87450787, \text{ approximately, or about } 15.87.$$