MATH-1140 (DUPRÉ) SPRING 2013 TEST 2 ANSWERS

DIRECTIONS: PRINT LAST NAME IN LARGE CAPITAL LETTERS IN THE UPPER RIGHT HAND CORNER OF EACH SHEET SUBMITTED, AND WRITE ALL FINAL ANSWERS CLEARLY IN THE SPACE PROVIDED FOR EACH QUESTION. WHEN FINISHED, TURN IN YOUR TEST PAPER FACE UP AND PLACE YOUR COVER SHEET FACE UP ON TOP OF YOUR TEST PAPER.

Suppose that X **and** Y **are unknowns with** $\mu_X = 12,\ \sigma_X = 4,\ \mu_Y = 23,$ **and** $\sigma_Y = 6.$ **Further suppose that the correlation coefficient giving the correlation of** X **with** Y **is** $\rho = .8.$ **Calculate the numerical values indicated for problems 1-9, using this information.**

1. The VARIANCE of X is $= \sigma_X^2 = 4^2 = 16.$

2. If a particular score for X is $x = 20$, then the equivalent standard score is

$$z_x = \frac{x - \mu_X}{\sigma_X} = \frac{20 - 12}{4} = 2.$$

3. If a particular standard score for Y is $z_y = 1.5$, then the actual equivalent raw score for Y is

$$y = \mu_Y + \sigma_Y z_y = 23 + (6)(1.5) = 23 + 9 = 32.$$

4. Given that a particular score for X is equivalent to the standard score $z_x = 2$, then, using the correlation of X with Y, the corresponding standard score that should be guessed for Y is the standard score

$$z_y = E(Z_Y|Z_X = 2) = \rho z_x = (.8)(2) = 1.6.$$

5. Given a particular score $x = 20$ for X, then using the correlation of X with Y, the corresponding score that should be guessed for Y is

$$y = E(Y|X = 20) = \mu_Y + \sigma_Y \rho \left[\frac{x - \mu_X}{\sigma_X}\right] = 23 + (6)(.8)(2) = 32.6.$$

6. Given that we DO NOT USE the correlation of X with Y to guess a value for Y, but simply guess $E(Y) = 23$, without looking to see the value of X, then our expected squared error is

$E(\text{error}^2) = \sigma_Y^2 = 6^2 = 36$

7. Given that we DO USE the correlation of X with Y to guess a value of Y from an observed value of X using linear regression properly, then our expected squared error is

$E(\text{error}^2|X \text{ value}) = (1-\rho^2)\sigma_Y^2 = (.36)(36) = 12.96.$

8. The COVARIANCE of X with Y is equal to

$Cov(X,Y) = \rho\sigma_X\sigma_Y = (.8)(4)(6) = 19.2.$

9. The VARIANCE of $X - Y$ is equal to

$Var(X-Y) = \sigma_X^2 + \sigma_Y^2 - 2Cov(X,Y) = 16 + 36 - (2)(19.2) = 52 - 38.4 = 13.6.$

Suppose that A and B are events with $P(A) = .6$, $P(B) = .4$, and $P(A|B) = .5$. Calculate the indicated probabilities in problems 10-14, using this information.

10. $P(A \text{ and } B) = P(A\&B) = P(A|B)P(B) = (.5)(.4) = .2.$

11. $P(A \text{ or } B) = P(A) + P(B) - P(A\&B) = .6 + .4 - .2 = .8$

12. $P(B \text{ but not } A) = P(A) - P(A\&B) = .4 - .2 = .2.$

13. $P(\text{neither } A \text{ nor } B) = 1 - P(A \text{ or } B) = 1 - .8 = .2$

14. $P(A \text{ or } B \text{ but not both}) = P(A \text{ or } B) - P(A\&B) = .8 - .2 = .6.$

15. The events A and B are [**CIRCLE AT LEAST ONE**]

MUTUALLY INDEPENDENT

DO NOT CIRCLE THIS: In order for the events A and B to be Mutually Independent, it is necessary that $P(A|B) = P(A)$, but $.6 \neq .5$, so they cannot be mutually independent.

MUTUALLY EXCLUSIVE

DO NOT CIRCLE THIS: In order for the events A and B to be Mutually Exclusive, it is necessary that $P(A\&B) = 0$, but $P(A\&B) = .2 \neq 0$, so they cannot be mutually exclusive.

NEITHER INDEPENDENT NOR EXCLUSIVE
ANSWER: [CIRCLE THIS ONE]

Suppose that we have twenty cards from a standard deck of cards so as to have 5 of each suit. Suppose that we deal out four cards from this deck of twenty cards.

16. How many ways can this be done so that all four cards are of the same suit?

ANSWER: To make such a hand with all four cards of the same suit, we can first choose the suit and then select four of the five cards of that suit, so

$$\text{Number of Ways } = C(4,1)C(5,4) = (4)(5) = 20.$$

17. How many ways can this be done so that there are three spades and one heart?

ANSWER: To make a hand with three spades and one heart, first choose 3 of the 5 spades and then choose one of the 5 hearts, so

$$\text{Number of Ways } = C(5,3)C(5,1) = (10)(5) = 50.$$