MATH-1140 (DUPRÉ) SPRING 2014 TEST 1 ANSWERS

Suppose that A, B, and C are statements with $P(A) = .6,\ P(B) = .7$, and $P(C) = .4$. Suppose $P(A\&B) = .5$ and $P(A$ or B or $C) = .9$. Then

4 and 5. $P([A\&C]$ or $[B\&C]) = P([A$ or $B]\&C) =$

$$= P(A \text{ or } B) + P(C) - P(A \text{ or } B \text{ or } C)$$

$$= P(A) + P(B) - P(A\&B) + P(C) - P(A \text{ or } B \text{ or } C)$$

$$= .6 + .7 - .5 + .4 - .9 = .3$$

FINAL ANSWER: .3

Suppose that a box contains 5 BLUE blocks, 3 RED blocks, and 2 GREEN blocks. Suppose that four blocks will be drawn from the box without replacement one after another.

6. What is the probability that the first block drawn will be red?

FINAL ANSWER: .3

7. What is the probability that the third block drawn will be RED given that the first will be GREEN and the second BLUE?

FINAL ANSWER: 3/8

8. What is the probability that the first block drawn will be RED given that the second will be BLUE and the third RED?

FINAL ANSWER: 2/8

9. What is the probability that two of the drawn blocks will be BLUE and two will be RED?

$$P(\text{2BLUE and 2 RED} \mid \text{four chosen}) = \frac{C(5,2) \cdot C(3,2)}{C(10,4)} = \frac{(10)(3)}{210} = \frac{1}{7} = .1428571429$$

FINAL ANSWER: 1/7 or .143 or $C(5,2)C(3,2)/C(10,4)$

Suppose in addition to the preceding information, that BLUE blocks are worth ONE dollar, that RED blocks are worth TEN dollars and green blocks are worth TWENTY dollars.

10. What is the total worth of the blocks in the box?

FINAL ANSWER: 75

11. If X is the value of the first block drawn, then what is $E(X)$?

Since there are ten blocks in the box and none are preferred as more likely to be drawn, each is expected to be worth one tenth of the total value in the box, or 7.5.

FINAL ANSWER: 7.5

12. If W is the value of the third block drawn, then what is $E(W)$?

Since there are ten blocks in the box and none are preferred as more likely to be drawn, each is expected to be worth one tenth of the total value in the box, or 7.5.

FINAL ANSWER: 7.5

13. If T is the total value of the four blocks drawn, then what is $E(T)$?

Since there are ten blocks in the box and none are preferred as more likely to be drawn, each is expected to be worth one tenth of the total value in the box, or 7.5. Therefore, four of them should be expected to be worth four times as much as a single block, or 30.

FINAL ANSWER: 30

Suppose that X is an unknown which has the possible values $-1, 0, 2, 3$, with probabilities

$$P(X = -1) = .2,\ P(X = 0) = .4,\ P(X = 2) = .3,\ P(X = 3) = .1.$$

14. What is the probability that X is at most 2?

$$P(X \text{ is at most } 2) = P(X \leq 2) = 1 - P(X = 3) = 1 - .1 = .9.$$

FINAL ANSWER: .9

15. What is the expected value of X?

Remember, our rules dictate that to calculate $E(X)$ you must sum all products obtained by multiplying values by their probabilities. Thus

$$E(X) = (-1)(.2) + (0)(.4) + (2)(.3) + (3)(.1) = -.2 + .6 + .3 = .7.$$

FINAL ANSWER: .7

16. What is the expected value of X^2?

$$E(X^2) = (-1)^2(.2) + (0)^2(.4) + (2)^2(.3) + (3)^2(.1) = .2 + 1.2 + .9 = 2.3.$$

FINAL ANSWER: 2.3

17. What is the variance of X?

Even though variance is defined (using the notation $\mu_X = E(X)$)

$$Var(X) = E([X - \mu_X]^2),$$

we observed that using our rules this simplifies to

$$Var(X) = E(X^2) - \mu_X^2$$

which here gives

$$Var(X) = 2.3 - (.7)^2 = 1.81.$$

FINAL ANSWER: 1.81

Suppose that we are writing a string of letters in line (that is a "word") taken from the alphabet A, B, C, D, K, L, M.

18. How many words of length 5 letters are possible if the same letter can be used more than once?

If the same letter can be used more than once, for each position there are 7 possible letters which can be put in that position, and a 5 letter word has 5 positions, so that means there are 7^5 possible words, and

$$7^5 = 16807.$$

FINAL ANSWER: 16807 or 7^5

19. How many 5 letter words are possible if all the letters in the word must be different?

If all the letters must be different, then the word is an arrangement of 5 of the 7 available letters, so there are $P(7,5)$ words with all letters different.

$$P(n,k) = \frac{n!}{(n-k)!},$$

so

$$P(7,5) = \frac{7!}{2!} = 2520.$$

FINAL ANSWER: 2520 or $P(7,5)$

20. How many 10 letter words are possible which have three A's, four B's and three K's?

There are $C(10;3,4,3)$ ten letter words having three A's, four B's, and three K's, where

$$C(10;3,4,3) = \frac{10!}{3!4!3!} = 4200.$$

FINAL ANSWER: 4200 or $C(10;3,4,3)$ or $\frac{10!}{3!4!3!}$