MATH-1140 (DUPRÉ) SPRING 2014 TEST 2 ANSWERS

Suppose that X and Y are unknowns with $\mu_X = 10,\ \sigma_X = 2,\ \mu_Y = 15,$ and $\sigma_Y = 4$. Further suppose that the correlation coefficient giving the correlation of X with Y is $\rho = .8$. Calculate the numerical values indicated for problems 1-9, using this information.

1. The VARIANCE of X is equal to

$$Var(X) = \sigma_X^2 = 2^2 = 4.$$

FINAL ANSWER: 4

2. If a particular score for X is $x = 13$, then the equivalent standard score is

$$z_x = \frac{x - \mu_X}{\sigma_X} = \frac{13 - 10}{2} = \frac{3}{2} = 1.5$$

FINAL ANSWER: $\frac{3}{2}$ or 1.5

3. If a particular standard score for Y is $z_y = 2.3$, then the actual equivalent raw score for Y is

$$y = \mu_Y + z_y \cdot \sigma_Y = 15 + (2.3)(4) = 15 + 9.2 = 24.2$$

FINAL ANSWER: 24.2

4. Given that a particular score for X is equivalent to the standard score $z_x = 3$, then, using the correlation of X with Y, the corresponding standard score that should be guessed for Y is the standard score

$$z_y = E(Z_Y|Z_X = 3) = \rho \cdot z_x = (.8)(3) = 2.4$$

FINAL ANSWER: 2.4

5. Given a particular score $x = 12$ for X, then using the correlation of X with Y, the corresponding score that should be guessed for Y is

$$y = E(Y|X = 12) = \mu_Y + \rho \cdot z_x \cdot \sigma_Y = \mu_Y + \frac{\rho \cdot \sigma_Y}{\sigma_X}(x - \mu_X) = 15 + (1.6)(2) = 15 + 3.2 = 18.2$$

FINAL ANSWER: 18.2

6. Given that we DO NOT USE the correlation of X with Y to guess a value for Y, but simply guess $y = 15$, without looking to see the value of X, then our expected squared error is equal to

$$Var(Y) = \sigma_Y^2 = 4^2 = 16.$$

FINAL ANSWER: 16

7. Given that we DO USE the correlation of X with Y to guess a value of Y from an observed value of X using linear regression properly, then our expected squared error is equal to

$$E(\text{squared error}|\text{properly use linear regression})$$
$$= (1 - \rho^2) \cdot \sigma_Y^2 = (1 - .64)(16) = (.36)(16)$$
$$= [(.6)(4)]^2 = (2.4)^2 = 5.76.$$

FINAL ANSWER: 5.76

8. The COVARIANCE of X with Y is equal to

$$Cov(X, Y) = \rho \cdot \sigma_X \cdot \sigma_Y = (.8)(2)(4) = 6.4.$$

FINAL ANSWER: 6.4

9. The VARIANCE of $X - Y$ is equal to

$$Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y) = 4 + 16 - 2(6.4) = 20 - 12.8 = 7.2.$$

FINAL ANSWER: 7.2

Suppose that A and B are events with $P(A) = .4$, $P(B) = .3$, and $P(A|B) = .8$. Calculate the indicated probabilities in problems 10-14, using this information.

10. $P(A \text{ and } B) = P(A|B) \cdot P(B) = (.8)(.3) = .24$

FINAL ANSWER: 0.24

11. $P(A \text{ but not } B) = P(A) - P(A\&B) = .4 - (.24) = .16$

FINAL ANSWER: 0.16

Suppose that we have a batch of ten widgets of which four are defective. We randomly select two widgets (without replacement) and check each to see if it is defective.

12. What is the probability that both are defective?

$$P(\text{both defective}) = \frac{C(4,2)}{C(10,2)} = \frac{6}{45} = \frac{2}{15}.$$

FINAL ANSWER: 2/15

13. What is the probability that one is defective and one is not?

$$P(\text{one defective and one not}) = \frac{C(4,1)C(6,1)}{C(10,2)} = \frac{4 \cdot 6}{45} = \frac{8}{15}.$$

FINAL ANSWER: 8/15

Specify the mathematically precise distribution of X (Choices: hypergeometric, geometric, binomial, exponential, Poisson, normal),

14. if X is the length of a fish in inches given the population mean is 12 and the standard deviation is 3.

ANSWER: When you only know the mean and standard deviation, the distribution must be normal.

FINAL ANSWER: NORMAL

15. if X is the number of voters in a simple random sample of size 20 of the population of voters in Smallville (population 200), who say they will support the current mayor for reelection.

ANSWER: Simple random sampling means without replacement, so as the population is small, the success count must be hypergeometric.

FINAL ANSWER: HYPERGEOMETRIC

16. if X is the number of speeders caught by a radar speed trap out of 100 passing cars, assuming that the cars speeds are independent of one another and that ten percent of the cars are speeding.

ANSWER: This is counting successes with independent trials, so the success count is binomial.

FINAL ANSWER: BINOMIAL

17. if X is the number of bears in a given 10 square mile parcel of forest in a 10000 square mile forest where the number of bears in disjoint regions of forest is assumed independent and there are on average 2 bears per square mile.

ANSWER: This is counting successes with a continuous sample size disjoint regions having independent success counts, so the distribution is the Poisson distribution.

FINAL ANSWER: POISSON

18. if X is the number of times we need to catch a red fish before finding one over 12 inches long (assuming we throw each fish back which is too short).

ANSWER: This is waiting for the first success with independent trials, so the distribution is geometric.

FINAL ANSWER: GEOMETRIC

19. if X is the number of square miles of forest we need to search to find the first bear in a 10000 square mile forest where the number of bears in disjoint regions of forest is assumed independent and there are on average 2 bears per square mile.

ANSWER: This is waiting for the first success with a continuous measure of sampling size and with disjoint regions having independent success counts, so the distribution is exponential.

FINAL ANSWER: EXPONENTIAL

20. if X is the number of cars whose speeds we check in order to find the first speeding car, assuming that the cars speeds are independent of one another, and that ten percent of the cars are speeding.

ANSWER: This is waiting for the first success with independent trials, so the distribution is geometric.

FINAL ANSWER: GEOMETRIC