

MATH-1150 (DUPRÉ) FALL 2011 PRACTICE TEST 3

**FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.**

**SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.**

**THIRD: WRITE YOUR SPRING 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.**

**FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.**

1. If the line  $L$  in the plane with rectangular coordinates has slope 3 and passes through the point  $(5, 4)$ , then what is the equation of  $L$ ?

2. If the line  $L$  in the plane passes through the points  $(2, 4)$  and  $(5, 10)$ , then what is the slope of  $L$ ?

3. If the line  $L$  in the plane passes through the points  $(2, 4)$  and  $(5, 10)$ , then what is the equation of  $L$ ?

4. What is the distance from the point  $(2, 4)$  to the point  $(5, 8)$ ?

5. What is the equation of the circle in the rectangular coordinate plane with center at  $(2, 4)$  having radius 3?

6. If a bee is flying along the line  $y = 3(x - 5) + 4$  so that its  $x$ -coordinate increases at the rate of 2 cm/sec, then what is the rate of increase of its  $y$ -coordinate?

7. If the line  $y = 3(x - 5) + 4$  is tangent to the curve  $C$  in the plane at the point  $(5, 4)$  and a bee is flying along  $C$  and at the instant the bee passes  $(5, 4)$  its  $x$ -coordinate increases at the rate of 2 cm/sec, then what is the rate of increase of its  $y$ -coordinate at that same instant?

8. Suppose that an oil spill in the Gulf of Mexico has a boundary which is 200 miles long. The spill is partially contained but along one 20 mile length the spill breaks through the containment and the boundary moves outward at the rate of .07 miles/day, and along another 30 mile length the spill breaks through its containment and there the boundary moves outward at the rate of .04 miles/day. What is the total rate of increase of area of the oil spill due to these containment breaks in square miles per day?

9. Suppose that  $f$  is the function whose value at  $x$  is given by  $f(x) = \sqrt{4 - x^2}$ . What is the domain of  $f$ ?

10. If  $f(x) = \sqrt{4 - x^2}$  and  $g(x) = \sqrt{3 + x^2}$ , and if  $h = f \circ g$ , then what is  $h(x)$ ?

11. If  $f(x) = \sqrt{4 - x^2}$  and  $g(x) = \sqrt{3 + x^2}$ , and if  $h = g \circ f$ , then what is  $h(x)$ ?

12. If  $f(x) = 1 + x^2$  with domain  $\{x \in \mathbb{R} : x \geq 0\}$ , and if  $g$  is the inverse function to  $f$ , then what is  $g(x)$ ?

13. If  $f$  and  $g$  are functions with  
 $f(2) = 5$ ,  $f(3) = 7$ ,  $f(5) = 4$ ,  $f(7) = 2$ ,  $g(2) = 4$ ,  $g(3) = 2$ ,  $g(5) = 7$ ,  $g(7) = 4$ , and if  $h = g \circ f$ , then what are  
 $(f + g)(3)$ ,  $h(3)$ ,  $(f \cdot g)(2)$ , and  $(f \cdot h)(3)$ ?

**14.** If lines  $L$  and  $M$  are parallel to each other and if  $L$  has slope 3, then what is the slope of  $M$ ?

**15.** If line  $L$  is parallel to the line with equation  $y = 7 - 4x$  and if  $(2, 3)$  is a point on line  $L$ , then what is the equation of  $L$ ?

**16.** If lines  $L$  and  $M$  are perpendicular to each other and if  $L$  has slope 3, then what is the slope of  $M$ ?

**17.** If line  $L$  is perpendicular to the line with equation  $y = 7 - 4x$  and if  $(2, 3)$  is a point on line  $L$ , then what is the equation of  $L$ ?

**18.** If  $f(x) = x^2 + 3x + 2$  what is the average rate of change of  $f$  from  $x = 0$  to  $x = 2$ ?

**19.** If  $f(x) = x^2 + 3x + 2$ , then what is the slope of the line through the two points on the graph of  $f$  where  $x = 0$  and  $x = 2$ ?

**20.** If  $f(x) = x^2 + 3x + 2$ , then what is the equation of the line through the two points on the graph of  $f$  where  $x = 0$  and  $x = 2$ ?

**21.** If  $f(x) = x^2 + 3x + 2$  and if  $a$  and  $b$  are two real numbers, then what is the slope of the line through the two points on the graph of  $f$  where  $x = a$  and  $x = b$ ?

**22.** If  $f(x) = x^2 + 3x + 2$ , what is  $f(x + a)$ ?

- 23.** If  $f(x) = x^2 + 3x + 2$ , what is  $f(x + b)$ ?
- 24.** If  $f(x) = x^2 + 3x + 2$ , what is  $f(x + b) - f(x)$ ?
- 25.** If  $f(x) = x^2 + 3x + 2$ , what is  $\frac{f(x + a) - f(x)}{a}$ ? What are the values when  $x = 2$  and  $a = .01, .001, .0001$ ?
- 26.** If  $f(x) = x^{3.4}$  what is  $f'(x)$ ?
- 27.** If  $f(x) = (1/2)x^2$ , what is the slope of the tangent line to the graph of  $f$  where  $x = 3$ ?
- 28.** If  $f(x) = (1/2)x^2$ , what is the equation of the tangent line to the graph of  $f$  where  $x = 3$ ?
- 29.** If  $f(x) = (1/2)x^2$ , what is the derivative of  $f$ ?
- 29.** If  $f(x) = (1/2)x^2$ , and  $y = f(x)$  what is  $dy/dx$ ?

Calculate the limits in the following problems.

$$30. \lim_{x \rightarrow 2} \frac{x^2 + 4}{x^3 + 2}$$

$$31. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

$$32. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 8}$$

$$33. \lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 + 2x - 8}$$

$$34. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 8}$$

$$35. \lim_{x \rightarrow 2^+} \frac{x^2 + 4}{x - 2}$$

$$36. \lim_{x \rightarrow 2^-} \frac{x^2 + 4}{x - 2}$$

$$37. \lim_{x \rightarrow 2} \frac{x^2 + 4}{(x - 2)(x - 3)}$$

$$38. \lim_{x \rightarrow 2^+} \frac{x^2 + 4}{(x - 2)(x - 3)}$$

$$39. \lim_{x \rightarrow 2^-} \frac{x^2 + 4}{(x - 2)(x - 3)}$$

$$40. \lim_{x \rightarrow \infty} \frac{7x^2 + 4}{(4x - 2)(5x - 3)}$$

$$41. \lim_{x \rightarrow -\infty} \frac{7x^2 + 4}{(4x - 2)(5x - 3)}$$

**Differentiate the given functions.**

$$42. f(x) = (7x^2 + 4)(4x - 2)(5x - 3)$$

$$43. f(x) = \frac{7x^2 + 4}{(4x - 2)(5x - 3)}$$

$$44. f(x) = \sqrt{7x^3 - 4x^2 + 3}$$

$$45. f(x) = (7x^2 + 4)^{99}$$

$$46. f(x) = \frac{(7x^2 + 4)^{99}}{(4x - 2)(5x - 3)}$$

**Suppose that  $f$  and  $g$  are functions, and that**

$$f(2) = 5, f'(2) = 4, g(2) = 3, g'(2) = 7, g(5) = 9, \text{ and } g'(5) = 11.$$

**Calculate the given limits and derivatives.**

$$47. h'(2) \text{ where } h = f - g.$$

$$48. h'(2) \text{ where } h = f \cdot g.$$

$$49. h'(2) \text{ where } h = f/g.$$

50.  $h'(2)$  where  $h = g \circ f$ .

51.  $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2}$

52.  $\lim_{x \rightarrow 2} \frac{f(x) - 5}{g(x) - 3}$

53.  $\lim_{x \rightarrow 2} f(x)$

54.  $\lim_{x \rightarrow 2} \frac{g(f(x)) - 9}{x - 2}$

55.  $\lim_{x \rightarrow 2} \frac{f(x)g(x) - 15}{x - 2}$

56. Suppose that a ladybug is walking along the curve with equation

$$x^3y^2 - x^2y^4 = 2.$$

At the instant the ladybug reaches the point  $(1, 1)$  the ladybug's  $x$ -coordinate is increasing at the rate of 3 centimeters per second. At that same instant what is the rate at which the ladybug's  $y$ -coordinate is increasing?

57. Given the equation

$$x^3y^2 - x^2y^4 = 2,$$

what is  $dy/dx$  in terms of  $x$  and  $y$ ?

58. Given the equation

$$x^3y^2 - x^2y^4 = 2,$$

what is the slope of the tangent line to the graph of the equation at the point  $(1, 1)$ ?

59. Given the equation

$$x^3y^2 - x^2y^4 = 2,$$

what is the equation of the tangent line to the graph of the equation at the point  $(1, 1)$ ?

60. For the function  $f(x) = x^3 + 3x^2 - 9x - 10$ , find all the critical points, all the inflection points, the largest intervals where  $f$  is increasing and likewise for decreasing, likewise for concave up and likewise for concave down. Sketch the curve.

61. For the function  $f(x) = (x + 1)^{2/3} - (x - 3)^{2/3}$ , find all the critical points.

62. Suppose that a balloon on a string is rising. However, the balloon's string is not vertical because the balloon is also moving horizontally at the same time because of cross wind. In fact, at a certain instant the string is 10 feet long and the balloon is above a point 6 feet in horizontal distance from the strings attachment to ground. If we assume that the balloon's horizontal velocity is equal to the wind speed, what is the wind speed at the instant in question, if we know that the balloon rises at the rate of 7 feet per minute at this instant?

63. Suppose that a region in the plane has a boundary that is 20 centimeters long and the region is expanding at a certain instant because the boundary is moving out 3 cm per minute. How fast is the region's area increasing at this particular instant?

64. Suppose that a potato develops a blister on its surface. At a given instant, the area of the blister 3 square centimeters and the surface of the blister moves out from the potato at the rate of 2 centimeters per week. What is the rate of volume increase of the potato at this instant due to the blister?

65. If a rectangle has two of its edges on the coordinate axes, one vertex at the origin and the opposite vertex on the curve  $y = (x - 1)(x + 4)$ , what is the maximum area it can have and where is the vertex opposite the origin for the rectangle of maximum area.

66. If  $f(x) = 6x^3 + 12x^2$ , what is the area under the graph of  $f$  between  $x = 1$  and  $x = 2$ ?

67. if  $f(x) = x^2 + x + 1$ , what is  $\sum_{k=2}^4 f(k)$ ?



**CALCULATE THE INTEGRALS INDICATED**

**68.**  $\int x^5 dx.$

**69.**  $\int [x^5 + x^3] dx.$

**70.**  $\int 7x^5 dx.$

**71.**  $\int [7x^5 - 2x^3] dx.$

**72.**  $\int \left[ \frac{7x^5 - 9x^3}{x^2} \right] dx.$

**73.**  $\int \sqrt{x} dx.$

**74.**  $\int \sqrt{7x} dx.$

**75.**  $\int \sqrt{x+3} dx.$

**76.**  $\int_2^4 x^5 dx.$

**77.**  $\int_2^4 [x^5 + x^3] dx.$

78.  $\int_2^4 7x^5 dx.$

79.  $\int_2^4 [7x^5 - 2x^3] dx.$

80.  $\int_2^4 \left[ \frac{7x^5 - 9x^3}{x^2} \right] dx.$

81.  $\int_2^4 \sqrt{x} dx.$

82.  $\int_2^4 \sqrt{7x} dx.$

83.  $\int_2^4 \sqrt{x+3} dx.$

84. If the function  $F$  is defined by

$$F(x) = \int_0^x [t^3 + \sqrt{t^3 + t^2 + 2}] dt,$$

then what is  $F'(1)$ ?

85. If the function  $F$  is defined by

$$F(x) = \int_0^{(x^2)} \sqrt{t} dt,$$

then what is  $F'(1)$ ?