MATH-1150 (DUPRÉ) FALL 2011 TEST 1 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR FALL 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.
NO CALCULATORS ALLOWED.

FIFTH: There are 14 problems. Each problem is worth 7 points except the last problem which is worth 9 points.

1. If the line L in the plane with rectangular coordinates has slope -5 and passes through the point $(-3, 4)$, then what is the equation of L?

$$y = 4 - 5(x - (-3))$$

2. If the line L in the plane passes through the points $(-2, 10)$ and $(2, 2)$, then what is the slope of L?

$$\text{slope of } L = \frac{10-2}{-2-2} = \frac{8}{-4} = -2$$

3. If the line L in the plane passes through the points $(-2, 10)$ and $(2, 2)$, then what is the equation of L?

$$y = 10 - 2(x - (-2)) \text{ or } y = 2 - 2(x - 2)$$

4. What is the distance from the point $(-2, 1)$ to the point $(4, 9)$?

$$\sqrt{(-2-4)^2 + (9-1)^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

5. What is the equation of the circle in the rectangular coordinate plane with center at $(-2, 1)$ having radius 5?

$$(x - (-2))^2 + (y - 1)^2 = 5^2 = 25$$

6. If the line L in the rectangular coordinate plane has slope 5 and if moving from point P on L to point Q on L increases the horizontal coordinate by 4 units, then how much is the vertical coordinate increased by moving from P to Q?

$$\Delta y = 5 \cdot \Delta x = 5 \cdot 4 = 20$$

7. If the line $y = 5(x - 3) + 4$ is tangent to the curve C in the plane at the point $(5, 4)$ and a bee is flying along C and at the instant the bee passes $(5, 4)$ its $x-$coordinate increases at the rate of 2 cm/sec, then what is the rate of increase of its $y-$coordinate at that same instant?

Using $\dot{z}$ to denote the rate of change of any quantity z which changes with time, we have

$$\dot{y} = 5 \cdot \dot{x} = 5 \cdot 2 = 10$$

8. Suppose that a drink spill on a table top, half a second after the initial spill, has a boundary which is 100 cm long. The spill is partially contained by napkins but half a second after the initial spill, along one 10 cm length the spill breaks through the containment and the boundary moves outward at the rate of 3 cm per second, and along another 15 cm length the spill breaks through its containment and there the boundary moves outward at the rate of 2 cm per second. At the instant half a second after the initial spill, what is the total rate of increase of area of the drink spill due to these containment breaks in square cm per second?

Using $\dot{z}$ to denote the rate of change of any quantity z which changes with time, and letting A denote the area of the spill which then changes with time,

$$\dot{A} = L_1 v_1 + L_2 v_2 = 10 \cdot 3 + 15 \cdot 2 = 30 + 30 = 60 \text{ sq cm per sec}$$

9. Suppose that f is the function whose value at x is given by $f(x) = \sqrt{4 + x}$. What is the domain of f?

The quantity inside the radical cannot be negative, so the domain is given by the inequality

$$x + 4 \geq 0 \text{ or equivalently } x \geq -4$$

10. If $f(x) = \sqrt{4 + x}$ and $g(x) = 3 + x^2$, and if $h = f \circ g$, then what is $h(x)$?

$$h(x) = f(g(x)) = \sqrt{4 + (3 + x^2)}$$

11. If $f(x) = \sqrt{4 + x}$ and $g(x) = 3 + x^2$, and if $h = g \circ f$, then what is $h(x)$?

$$h(x) = g(f(x)) = 3 + (\sqrt{4 + x})^2$$

12. If f is the function with rule $f(x) = 2x + 5$ and with domain $\{x \in \mathbb{R} : x \geq 0\}$, and if g is the inverse function to f, then what is the domain of g and what is $g(10)$?

The equation $y = g(x)$ is the equation $x = 2y + 5$ or $y = (1/2)(x - 5)$. The domain of f is $x \geq 0$, which for the equation of $y = g(x)$ means that $y \geq 0$ but since $y = (1/2)(x - 5)$, this is the inequality $(1/2)(x - 5) \geq 0$, or $x \geq 5$. Therefore the domain of g is the set of numbers x with $x \geq 5$. Also, $g(10) = (1/2)(10 - 5) = 5/2$.

13. If line L is parallel to the line with equation $y = 7 - 4x$ and if $(2, 3)$ is a point on line L, then what is the equation of L?

Since L is parallel to the line $y = 7 - 4x$, it must have slope -4, so in point-slope form the equation is

$$y = 3 - 4(x - 2).$$

14. If $f(x) = x^2$, then what is the actual numerical value of the slope of the line through the two points on the graph of f where $x = a$ and $x = b$ if $a = 2.7$ and $b = 2.703$? (Hint: use $a = 2.7$ and $b = a+h$, expressing the slope in terms of a and h, simplify the resulting expression for the slope as much as possible, and then set $a = 2.7$ and $h = .003$ in order to calculate the final numerical value of the slope.)

If we denote the slope by m, then

$$m = \frac{f(b) - f(a)}{b - a} = \frac{b^2 - a^2}{b - a} = \frac{(b - a)(b + a)}{b - a} = a + b = 2.7 + 2.703 = 5.403.$$

Of course this method requires you to be able to factor the difference of two squares. Using the hint, instead we replace $b = a + h$ so $b - a = h$ and then

$$\begin{aligned} m = \frac{f(b) - f(a)}{b - a} = \frac{f(a + h) - f(a)}{h} &= \frac{(a + h)^2 - a^2}{h} \\ &= \frac{a^2 + 2ah + h^2 - a^2}{h} \\ &= \frac{2ah + h^2}{h} \\ &= 2a + h \\ &= a + b \\ &= 5.403. \end{aligned}$$

Notice that even though this latter method seems to have more symbols involved, and seems to take longer, it does not require us to figure out the factorization in order to get the cancellation with the denominator. You just simplify the numerator and cancel. For instance, for the same problem with f replaced by the function g, where $g(x) = x^5$, you would have to factor $b^5 - a^5$, whereas on making the substitution $b = a + h$, we have after multiplying out $(a + h)^5$ and collecting like terms (which of course though tedious is straight formward),

$$(a + h)^5 = a^5 + 5a^4h + 10a^3h^2 + 10a^2h^3 + 5ah^4 + h^5$$

and therefore

$$\frac{g(b) - g(a)}{b - a} = \frac{5a^4h + 10a^3h^2 + 10a^2h^3 + 5ah^4 + h^5}{h} = 5a^4 + 10a^3h + 10a^2h^2 + 5ah^3 + h^4.$$

From this we see in particular that on replacing h by $b - a$, that it must be the case that the factorization of $b^5 - a^5$ is

$$b^5 - a^5 = (b - a)(5a^4 + 10a^3(b - a) + 10a^2(b - a)^2 + 5a(b - a)^3 + (b - a)^4).$$

In general, for polymomials of degree 5 and more there is no general factorization formula, due to a famous ninetheenth century theorem of the mathematician Evarist Galois.