MATH-1150 (DUPRÉ) FALL 2011 TEST 2 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR FALL 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

Suppose $(2,5)$ **is a point on the graph of the function** f **and the line** $y = 4(x-2)+5$ **is tangent to the graph of** $y = f(x)$ **at the point** $(2,5)$. **THEN**

1. $f(2) = 5$

2. $f'(2) = 4$

3. $\lim_{x \to 2} f(x) = 5$

4. $\lim_{x \to 2} \dfrac{f(x) - 5}{x - 2} = 4$

5. $\lim_{x \to 2} \left[f(x) \cdot \dfrac{f(x) - 5}{x - 2} \right] = 5 \cdot 4 = 20$

EXPLANATION FOR ANSWERS: The fact that $(2,5)$ is on the graph of f tells us that $f(2) = 5$, giving the answer to question number 1. The fact that the graph of $y = f(x)$ has a tangent line at $(2,5)$ tells us that f is differentiable at $x = 2$. The fact that the tangent line at $(2,5)$ has slope $m = 4$ tells us that $f'(2) = 4$, which gives the answer to second question, question number 2. The fact that f is differentiable at $x = 2$ implies that also f is continusous at $x = 2$, and this tells us the answer to question number 3 is simply $f(2) = 5$, using the answer to question number 1. By the definition of the derivative, we know that the answer to question 4 is $f'(2)$, but from the answer to question 2, we know this to be 4, so the answer to question number 4 must also be 4. Obviously, by the product rule for limits, the answer to question number 5 is the product of the answers to the previous two questions, which is simply $5 \cdot 4 = 20$. It cannot get much easier.

Suppose that the area of a lake is increasing due to a ten mile length of white sand beach which is eroding at the rate of 3 miles per century and another five mile length of black sand beach which is eroding at the rate of 2 miles per century.

6. What is the rate at which the area of the lake is increasing due to the white sand beach erosion in square miles per century?

The erosion is causing the boundary of the lake to move out in the normal direction at the stated rates along each beach. Remember, any time a region increases in area due to a moving boundary, the rate of increase of area due to the moving boundary is

$$\frac{dA}{dt} = L \cdot v_{out},$$

where v_{out} is the normal velocity along the moving boundary. Thus the white sand beach erosion is causing the lake to increase area at the rate of

$$\frac{dA}{dt} = (10)(3) = 30 \text{ square miles per century.}$$

7. What is the rate at which the area of the lake is increasing due to the black sand beach erosion?

Likewise,

$$\frac{dA}{dt} = (5)(2) = 10 \text{ square miles per century.}$$

8. What is the rate at which the area of the lake is increasing due to the erosion of the two beaches?

Obviously, the rate at which the area of the lake is increasing due to the erosion of both beaches is the sum of the individual rates of increase due to the erosion of each beach alone,

$$\frac{dA}{dt} = 30 + 30 = 40 \text{ square miles per century.}$$

In the following problems find the indicated derivatives.

9. $\dfrac{dy}{dx}$ where $y = x^7 + 2x^3$

Using the sum, constant multiple, and power rules for differentiation, we have

$$\frac{dy}{dx} = 7x^6 + 2 \cdot 3x^2.$$

10. $f'(x)$ where $f(x) = x^{-4/5}$

Since the power rule works for any real number exponent or power,

$$f'(x) = (-4/5)x^{-9/5}.$$

11. $\dfrac{dy}{dx}$ where $y = \dfrac{x^3 + 4}{x^5 - 8}$

We just use the quotient rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2},$$

so here,

$$\frac{dy}{dx} = \frac{3x^2(x^5 - 8) - (x^3 + 4)5x^4}{(x^5 - 8)^2}.$$

12. $\dfrac{dy}{dx}$ where $y = (x^5 + 3x^4 + 2)^{1/3}$

Here we must use the chain rule for differentiating a composite function, that is the chain rule for differentiating a function of a function. The chain rule says $(f \circ g)' = [f' \circ g] \cdot g'$. That is,

$$(f \circ g)'(x) = [f(g(x)]' = f'(g(x)) \cdot g'(x),$$

or with $u = g(x)$ and $y = f(u)$,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

That is, we can think that the du's cancel out.

So here, $y = f(u) = u^{1/3}$, and $u = g(x) = x^5 + 3x^4 + 2$, and the result is

$$\frac{dy}{dx} = (1/3)u^{-2/3} \cdot (5x^4 + 12x^3) = (1/3)(x^5 + 3x^4 + 2)^{-2/3}(5x^4 + 12x^3).$$

Suppose f and g are functions with

$$f(3) = 4,\ f'(3) = 7,\ f(5) = 6,\ f'(9) = 2,\ g(3) = 9,\ g'(3) = 5,\ g'(4) = 8,\ g(7) = 11.$$

These last three problems just use the differentiation rules.

13. If $h = f - g$ then $h'(3) = 2$

By the sum and difference rules for derivatives, the derivative of a difference is simply the difference of the derivatives:

$$h'(3) = f'(3) - g'(3) = 7 - 5 = 2.$$

14. If $h = f \cdot g$ then $h'(3) = 83$

By the product rule for derivatives,

$$h'(3) = f'(3) \cdot g(3) + f(3) \cdot g'(3) = 7 \cdot 9 + 4 \cdot 5 = 63 + 20 = 83.$$

15. If $h = f \circ g$, then $h'(3) = 10$

By the chain rule for differentiation:

$$h'(3) = f'(g(3)) \cdot g'(3) = f'(9) \cdot g'(3) = 2 \cdot 5 = 10.$$