MATH-1150 (DUPRÉ) SPRING 2011 TEST 2 ANSWERS

Wednesday 30 March 2011

DIRECTIONS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR CORRECT SPRING 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

FIFTH: Any failure to follow any part of any of the above directions can result in additional loss of credit.

1. If $f(x) = x^5$, then what is $f'(x)$ the derivative of f at x?

$$f'(x) = 5x^4$$

2. If $y = t^4 - 5t^2 + 3$, then what is dy/dt in terms of t?

$$\frac{dy}{dt} = 4t^3 - 5 \cdot 2t$$

3. If $z = e^x$, then what is dz/dx?

$$\frac{dz}{dx} = e^x$$

4. Differentiate but **DO NOT** simplify $f(x) = (e^x)\sin x$.

$$f'(x) = e^x \sin x + e^x \cos x$$

5. Differentiate but **DO NOT** simplify $f(x) = \dfrac{x^3 + e^x}{e^x + x^5}$.

$$f'(x) = \frac{(3x^2 + e^x)(e^x + x^5) - (x^3 + e^x)(e^x + 5x^4)}{(e^x + x^5)^2}$$

6. If both u and v are differentiable functions of time t, and if

$$e^{u^3} + \sin(u + v) = \cos v,$$

then differentiate both sides of the equation to find an equation relating $u, v, du/dt$, and dv/dt.

$$e^{u^3} \cdot 3u^2 \frac{du}{dt} + [\cos(u + v)] \left[\frac{du}{dt} + \frac{dv}{dt}\right] = [-\sin v] \frac{dv}{dt}$$

7. Differentiate but **DO NOT** simplify $g(x) = \ln(\sec x + \tan x)$.

$$g'(x) = \frac{1}{\sec x + \tan x} [(\sec x)(\tan x) + \tan^2 x]$$

8. Give the equation of the tangent line to the curve having equation

$$y = \sqrt{x}$$

at the point $(25, 5)$, that is the point where $x = 25$ and $y = 5$.

ANSWER: The tangent line to the curve $y = \sqrt{x}$ has slope m which is the value of dy/dx at the point where $x = 25$. Since

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-1/2}) = (-1/2)x^{-1/2} = \frac{1}{2\sqrt{x}}$$

it follows that

$$m = \frac{1}{2\sqrt{25}} = \frac{1}{2 \cdot 5} = \frac{1}{10},$$

and therefore **the equation of the tangent line** is

$$y - 5 = \frac{1}{10}(x - 25).$$

9. For the curve with equation

$$x^2 - xy + y^3 = 3,$$

find the slope of the tangent line at the point $(2,1)$, that is the point where $x = 2$ and $y = 1$. Your final answer should be a number expressed in terms of the numbers 1 and 2, but you do not need to simplify or calculate. However, you will lose credit for each appearance of the symbols x or y in your final answer.

ANSWER: Using implicit differentiation, we can differentiate both sides of the equation with respect to x to get an equation relating x, y, and dy/dx. That result is

$$2x - \left(1 \cdot y + x \cdot \frac{dy}{dx}\right) + 3y^2 \frac{dy}{dx} = 0.$$

We can now put $x = 2$ and $y = 1$ in the above equation getting

$$2 \cdot 2 - \left(1 \cdot 1 + 2 \cdot \frac{dy}{dx}\right) + 3 \cdot 1^2 \cdot \frac{dy}{dx} = 0,$$

or

$$4 - 1 - 2\frac{dy}{dx} + 3\frac{dy}{dx} = 0,$$

which now simplifies to

$$3 + \frac{dy}{dx} = 0.$$

This is now easily solved for dy/dx giving the **slope**

$$\frac{dy}{dx} = -3.$$

10. At exactly noon when and where the Sun is directly overhead, a helium ballon 8 feet off the ground is being held by a 10 foot length of string tied to a stake in the ground and is gaining altitude at the rate of 2 feet per minute. At that time (noon), assuming the string is stretched to be straight, how fast is the shadow of the balloon moving towards the stake in feet per minute? Your final answer must be an algebraic expression involving only numbers if you cannot calculate it.

ANSWER: We can consider the plane of the string, the vertical line segment from the balloon to its shadow, and the horizontal line segment from the balloon's shadow to the stake, and notice that the string is then the hypoteneuse of a right triangle formed with these two line segments. Let x be the length (in feet) of the horizontal line segment and let y be the length (in feet) of the vertical line segment. Since the string is 10 feet long, we have

$$x^2 + y^2 = 100.$$

Since the balloon at noon is gaining altitude at the rate of 2 feet per minute we have

$$\frac{dy}{dt} = 2 \text{ at noon.}$$

At noon, the ballon is 8 feet off the ground, so $y = 8$ at noon and therefore at noon,

$$x^2 + 8^2 = 100,$$

or

$$x = 6 \text{ at noon.}$$

On the other hand, since the string cannot change its length with time, we can differentiate the equation $x^2 + y^2 = 100$ with respect to time t and find

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0,$$

or, cancelling 2 from both sides of the equation,

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0.$$

At noon this gives us

$$6\frac{dx}{dt} = -8 \cdot 2,$$

or

$$\frac{dx}{dt} = -\frac{8 \cdot 2}{6} = -\frac{8}{3} \text{ feet per minute.}$$

This means that at noon, x is decreasing, that is **the balloon's shadow is getting closer to the stake at the rate of 8/3 feet per minute.**