MATH-1150 (DUPRÉ) SPRING 2011 QUIZ 3 ANSWERS

Wednesday 29 February 2012

DIRECTIONS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR CORRECT SPRING 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

FIFTH: Any failure to follow any part of any of the above directions can result in additional loss of credit.

Suppose $f, g,$ and h are functions with

$$f(2) = 4,\ g(2) = e,\ f'(2) = 5,\ g'(2) = 3,\ g(4) = 5 \ \textbf{ and } g'(4) = 9.$$

1. If $h = f \cdot g$, then $h'(2) = f'(2)g(2) + f(2)g'(2) = 5 \cdot e + 4 \cdot 3 = 5e + 12$.

2. If $h = f/g$, then $h'(2) = \dfrac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2} = \dfrac{5e - 12}{e^2}$.

3. If $h = g \circ f$, then $h'(2) = g'(f(2)) \cdot f'(2) = g'(4) \cdot f'(2) = 9 \cdot 5 = 45$.

4. If $h = f^e \cdot \ln[\pi + g]$ then $h'(2) = ([f^e]' \cdot \ln[\pi + g] + f^e \cdot [\ln(\pi + g)]')(2)$.
Now, $[f^e]' = ef^{e-1} \cdot f'$, so $[f^e]'(2) = e4^{e-1} \cdot 5$, and

$$[\ln[\pi + g]]' = \frac{[\pi + g]'}{\pi + g} = \frac{g'}{\pi + g},$$

so

$$[\ln[\pi + g]]'(2) = \frac{g'(2)}{\pi + g(2)} = \frac{3}{\pi + e}.$$

Therefore,

$$h'(2) = e4^{e-1} \cdot 5 \cdot \ln[\pi + e] + 4^e \cdot \frac{3}{\pi + e}.$$

5. If $w = f(x, y, z) = x^2 \cdot e^y \cdot \ln z$, and if we use over-dots to denote time rates of change, then express $\dot{w}$ in terms of x, $\dot{x}$, y, $\dot{y}$, z, and $\dot{z}$.

We can simply use the product rule here. We first view $w = x^2 \cdot e^y \cdot \ln z$ as being the product of two factors, the first being simply x^2 and the second being $e^y \ln z$. Then

$$\dot{w} = (x^2\dot{)}(e^y \ln z) + x^2(e^y \ln z\dot{)}.$$

Since $e^y \ln z$ is itself a product, we apply the product rule to it getting

$$(e^y \ln z\dot{)} = (e^y\dot{)} \ln z + e^y(\ln z\dot{)}.$$

By the chain rule, we have

$$(x^2\dot{)} = 2x\dot{x},$$

$$(e^y\dot{)} = e^y\dot{y},$$

and

$$(\ln z\dot{)} = \frac{1}{z} \cdot \dot{z} = \frac{\dot{z}}{z}.$$

Therefore

$$\dot{w} = (2x\dot{x})(e^y \ln z) + x^2 \left[(e^y\dot{y}) \ln z + e^y \frac{\dot{z}}{z} \right],$$

which suffices for the answer. We can notice that we can arrange the terms to have

$$\dot{w} = [2xe^y \ln z]\dot{x} + [x^2 e^y \ln z]\dot{y} + [x^2 e^y \cdot \frac{1}{z}]\dot{z}.$$

An alternate method is to get the coefficients for $\dot{x}$, $\dot{y}$, and $\dot{z}$ directly. We know the end result can be arranged in the form

$$\dot{w} = M(x, y, z)\dot{x} + N(x, y, z)\dot{y} + P(x, y, z)\dot{z},$$

for some coefficient functions M, N, P. The coefficient of $\dot{x}$ is obtained by differentiating $f(x, y, z)$ with respect to x keeping y and z constant. If we denote the operation of differentiating with respect to x keeping all other variables constant by ∂_x, then

$$M(x, y, z) = \partial_x[x^2 e^y \ln z] = 2xe^y \ln z.$$

Likewise, we can denote the operation of differentiating with respect to y keeping all other variables constant by ∂_y, and

$$N(x, y, z) = \partial_y[x^2 e^y \ln z] = x^2 e^y \ln z,$$

and similarly for z, we have

$$P(x, y, z) = \partial_z[x^2 e^y \ln z] = x^2 e^y \frac{1}{z}.$$

Therefore, again,

$$\dot{w} = [2xe^y \ln z]\dot{x} + [x^2 e^y \ln z]\dot{y} + [x^2 e^y \frac{1}{z}]\dot{z}.$$

Notice that with this second method, we only apply the product rule in its simplest form, the case where one of the factors is a constant. It is much easier using this second method.