MATH-1150 (DUPRÉ) SPRING 2011 TEST 1 ANSWERS

Wednesday15 February 2012

DIRECTIONS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR CORRECT SPRING 2011 MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOU FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

FIFTH: Any failure to follow any part of any of the above directions can result in additional loss of credit.

Suppose $f, g,$ and h are functions with

$$f(2) = 3,\ g(2) = 7,\ f'(2) = 4,\ g'(2) = -8, \quad \textbf{and} \quad g'(3) = 5.$$

1. If $h = f \cdot g$, then $h'(2) = f'(2)g(2) + f(2)g'(2) = 4 \cdot 7 + 3 \cdot [-8] = 28 - 24 = 4.$

2. If $h = f/g$, then $h'(2) = \dfrac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2} = \dfrac{4 \cdot 7 - 3 \cdot [-8]}{7^2} = \dfrac{28 + 24}{49} = \dfrac{52}{49}.$

3. If $h = g \circ f$, then $h'(2) = g'(f(2)) \cdot f'(2) = g'(3) \cdot f'(2) = 5 \cdot 4 = 20.$

4. If $h = f^{\pi} \cdot \ln g$, then $h'(2) = \pi f(2)^{\pi - 1} f'(2) \cdot \ln[g(2)] + [f(2)]^{\pi} \cdot \dfrac{g'(2)}{g(2)}$

$$= \pi[3^{\pi-1}] \cdot 4 \cdot \ln 7 + 3^{\pi} \cdot \frac{[-8]}{7} = 3^{\pi}\left[\frac{4}{3} \cdot \ln 7 - \frac{8}{7}\right].$$

5. If $h = [g \circ f] \cdot e^{g}$, then $h'(2) = [g'(f(2)) \cdot f'(2)]e^{g(2)} + g(f(2)) \cdot e^{g(2)} \cdot g'(2) = 20e^7 + g(3)e^7 \cdot (-8)$

$$= e^7[20 - 8 \cdot g(3)].$$

6. If L is the line with equation $8x - 4y = 12$, then the slope of L is $= 2$.

Notice the equation is equivalent to the equation $y = 2x - 3$ which has clearly has slope $= 2$.

7. If $f(x) = x^3 - [\ln x] + e^x - \sqrt{x} + x^\pi$, then $f'(x) = 3x^2 - \dfrac{1}{x} + e^x - \dfrac{1}{2\sqrt{x}} + \pi x^{\pi - 1}$.

8. If T is the tangent line to the graph of the equation $y = x^2 - 2x - 2$ at the point $(3, 1)$,

then the slope of T is $= 4$.

For we can define the function f by $f(x) = x^2 - 2x - 2$ and then we know the slope m of the tangent line is given by $m = f'(3)$. But $f'(x) = 2x - 2$, therefore $f'(3) = 2 \cdot 3 - 2 = 4$.

9. If T is the tangent line to the graph of the equation $x^2 - xy + y^3 = 3$ at the point $(2, 1)$,

then an equation for T is:

To see this imagine we travel along the curve in time so that x and y depend on time, and let $\dot{x}$ and $\dot{y}$ denote their rates of change. Differentiating both sides of the equation with respect to time, we have at any time during the motion

$$0 = \dot{3} = (x^2)^{\dot{}} - (xy)^{\dot{}} + (y^3)^{\dot{}} = 2x\dot{x} - [\dot{x}y + x\dot{y}] + 3y^2\dot{y} = [2x - y]\dot{x} + [3y^2 - x]\dot{y},$$

or simply

$$[2x - y]\dot{x} + [3y^2 - x]\dot{y} = 0.$$

In particular, at the instant we pass through the point $(2, 1)$ the rates of change satisfy

$$3\dot{x} + \dot{y} = 0,$$

which means that an equation of the tangent line is

$$3(x - 2) + (y - 1) = 0.$$

10. If $f(z) = \dfrac{z^7 - e^{[3z^5+2]}}{z^3 + \ln z}$,

then $f'(z) = \dfrac{[7z^6 - e^{[3z^5+2]} \cdot (3 \cdot 5z^4)][z^3 + \ln z] - [z^7 - e^{[3z^5+2]}][3z^2 + (1/z)]}{[z^3 + \ln z]^2}$