MATH-1150 DUPRE TEST II SPRING 2013 ANSWERS

MAURICE J. DUPRÉ

1. An eagle is coasting in a horizontal circle, by making use of updrafts. The circle has a radius of 50 feet and the eagle moves around this circle with a constant speed of 20 feet per second. What is the eagle's acceleration in feet per second per second?

ANSWER: The velocity vector is always tangent to the circle of motion and has constant length of 20. However, as the eagle travels the circle, his velocity vector changes and is not constant, only his speed is constant. For a circle of radius r travelled at speed v, the length of the acceleration vector is $a = v^2/r$, so here the acceleration of the eagle is $a = (20)^2/50 = 400/50 = 8$ feet per second per second. The direction of the acceleration vector is from the eagle's position to the center of the circle, so likewise, the acceleration vector is not constant, only its length, a, is constant.

2-6. Suppose that $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ are both smooth functions and that

$$f(5) = 3,\ g(5) = 2,\ f(2) = 4,\ g(3) = 7,\ f'(5) = 4,\ g'(3) = 2,\ f'(2) = 6,\ g'(5) = 10.$$

Calculate

$$(f - g)'(5) = f'(5) - g'(5) = 4 - 10 = -6$$

$$(fg)'(5) = f'(5)g(5) + f(5)g'(5) = (4)(2) + (3)(10) = 38$$

$$(f \circ g)'(5) = f'(g(5))g'(5) = f'(2)g'(5) = (6)(10) = 60$$

$$(g \circ f)'(5) = g'(f(5))f'(5) = g'(3)f'(5) = (2)(4) = 8$$

$$(f/g)'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{[g(5)]^2} = \frac{(4)(2) - (3)(10)}{2^2} = -\frac{11}{2}$$

7-9. Calculate $f'(x)$ if $f : \mathbb{R} \longrightarrow \mathbb{R}$ is

$$f(x) = \frac{x^5 - 8}{x^4 + 2x^2 + 3}.$$

$$f'(x) = \frac{(x^5 - 8)'(x^4 + 2x^2 + 3) - (x^5 - 8)(x^4 + 2x^2 + 3)'}{(x^4 + 2x^2 + 3)^2} = \frac{5x^4(x^4 + 2x^2 + 3) - (x^5 - 8)(4x^3 + 4x)}{(x^4 + 2x^2 + 3)^2}$$

Date: February 28, 2013.

10. Suppose $g : \mathbb{R} \longrightarrow \mathbb{R} \setminus \{0\}$ and $f : \mathbb{R} \longrightarrow \mathbb{R}$ are smooth functions with $f' = f$ and $g' = g$. If $h = (f/g)$, what is $h'(2)$?

ANSWER: Use the quotient rule for differentiation and the equations $f' = f$ and $g' = g$ giving

$$h' = \frac{f'g - fg'}{g^2} = \frac{fg - fg}{g^2} = \frac{0}{g^2} = 0.$$

Therefore $h'(x) = 0$ for any $x \in \mathbb{R}$, so in particular,

$$h'(2) = 0.$$

Department of Mathematics, Tulane University, New Orleans, LA 70118 USA
E-mail address, M. J. Dupré: mdupre@tulane.edu