MATH-1150 (DUPRÉ) TEST 1 SPRING 2013 ANSWERS

DIRECTIONS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

THIRD: WRITE YOUR CORRECT MATH-1150 SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

FOURTH: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

FIFTH: Any failure to follow any part of any of the above directions can result in additional loss of credit.

1 & 2 & 3 & 4. In the space below, draw a picture of two non-zero vectors which are not parallel to each other. Label one vector X and the other vector Y. As well, draw the pictures of $-Y$, $X+Y$, and $X-Y$. Also draw the picture of $(1/2)(X+Y)$ using the parallelogram picture for $X+Y$.

ANSWER: Once you have drawn the two vectors, simply reverse the arrow labeled Y and label it $-Y$ then draw X and Y with their tails together and complete the parallelogram. The diagonal arrow of the parallelogram with tail at the point where the tails of X and Y meet and tip at the opposite vertex of the parallelogram is $X+Y$. Now draw the arrow with tail at the tip of Y and tip at the tip of X making the other diagonal of the parallelogram. This arrow is a picture of $X-Y$. The point where the diagonals meet gives the tip of $(1/2)(X+Y)$ and the tail is again the same point of the parallelogram which coincides with the tails of X and Y.

5 & 6 & 7. A swimmer is swimming in a river. At a given instant, the swimmer is moving at 4 feet per second relative to the water. At that same instant, the water is moving at 3 feet per second relative to the Earth. Relative to the water, the direction the swimmer moves is in the direction from his feet to his head, which is the direction of his body. Give the swimmer's speed relative to the Earth in the following three cases for the direction of his body: with the water flow, against the water flow, and across the water flow.

ANSWER: The swimmer's velocity vector relative to the Earth is always the vector sum of his velocity relative to the water and the velocity of the water. Therefore when his body is in the direction of the flow, his speed relative to the Earth is the sum of his speed relative to the water and the speed of the water, as the velocity vectors point in the same direction, which is therefore 7 feet per second. When the swimmer's body direction is against the water flow, the velocity vectors point in opposite directions, and therefore in this case the swimmer's speed relative to Earth is $4-3=1$ foot per second. When the swimmer's body direction is across the water flow, then the two vectors are perpendicular, so the vector sum is an arrow which is the hypotenuse of the right triangle formed by the two vectors, so the speed is the hypotenuse length of a right triangle with sides of length 3 and 4, which is therefore 5 feet per second by the Pythagorean Theorem.

8 & 9. Paradise Island has a coast consisting of two beaches separated by stable rocky coast. One beach is 8 miles long and eroding at the rate of 3 miles per century and the other beach is 10 miles long and eroding at the rate of 2 miles per century. What is the rate of land loss in square miles per century due to the erosion along the 8 mile beach? What is the total rate of land loss due to erosion of both beaches combined?

ANSWER: The general principle here is that the rate of change of area due to a moving boundary is the length of the moving boundary multiplied by the outward or normal velocity of that moving boundary, assuming that all points of that moving boundary have the same normal velocity. If there are several parts of the boundary which move, then the total rate of change of area is the sum of the rates due to the individual pieces which move. Thus, the rate of land loss due to the 8 mile beach is $8 \cdot 3 = 24$ square miles per century. The rate of land loss due to the 10 mile beach is likewise $2 \cdot 10 = 20$ square miles per century and therefore the total rate of land loss due to the erosion of both beaches is $24 + 20 = 44$ square miles per century.

10. If $\dot{x} = 1$ and $y = 2x^3 - 5x^2 + 3$, then what is $\dot{y}$ in terms of x?

ANSWER: Using the rules for computing rates we know so far, together with $\dot{x} = 1$ we find

$$\dot{y} = 2 \cdot 3x^2\dot{x} - 5 \cdot 2x\dot{x} + 0 \cdot \dot{x} = 6x^2 - 10x.$$