MATH-1150 (DUPRÉ) SPRING 2013 TEST 3 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN INCLUDING THE PRINTED COVER SHEET.

SECOND: Write NEATLY and CLEARLY, putting your answers in the space provided. If I cannot read it you do not get credit.

PROBLEM 1. Make sure you follow the directions above, any failure results in loss of credit for this first problem on this test.

Suppose that the function $f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ is given by

$$f(x, y) = x^2y - xy^2 = xy(x - y).$$

We can easily see that

$$f(3, 2) = 6.$$

Therefore the point $(3, 2)$ is on the curve with equation $f(x, y) = 6$ which is a curve in the $(x, y)-$plane.

PROBLEM 2. Suppose that x and y are both functions of time, t, that $x = x(t)$, and $y = y(t)$, and the function $z = z(t)$ of time is given by $z = f(x, y)$, so that

$$z = x^2y - xy^2 = z(t) = f(x(t), y(t)) = [x(t)]^2y(t) - x(t)[y(t)]^2, t \in \mathbb{R}.$$

Use the chain rule to express $\dot{z}$ in terms of $x,\ y,\ \dot{x},\ \dot{y}$.

ANSWER: $\dot{z} = \left[2xy - y^2\right] \dot{x} + \left[x^2 - 2xy\right] \dot{y}$

PROBLEM 3. Suppose that a beetle is crawling along the graph of the curve $z = f(x, y) = 6$ so that the rate of increase of x for the beetle is 3 units per second at the instant the beetle passes through the point $(3, 2)$. What is the rate of increase of z for the beetle at the instant the beetle passes through the point $(3, 2)$?

ANSWER: For the beetle, z is constant with value 6 and therefore, for the beetle, $\dot{z} = 0$.

PROBLEM 4. Suppose that a beetle is crawling along the graph of the curve $z = f(x, y) = 6$ so that the rate of increase of x for the beetle is 3 units per second at the instant the beetle passes through the point $(3, 2)$. What is the rate of increase of y for the beetle at the instant the beetle passes through the point $(3, 2)$?

ANSWER: Since $\dot{z} = 0$ for the beetle, and as already we have

$$\dot{z} = [2xy - y^2]\dot{x} + [x^2 - 2xy]\dot{y},$$

in this equation we can set $x = 3,\ y = 2,\ \dot{x} = 3,\ \dot{z} = 0$, to get the equation

$$0 = [(2)(3)(2) - (2)^2](3) + [(3)^2 - (2)(3)(2)]\dot{y} = [12 - 4](3) - 3\dot{y},$$

so

$$3\dot{y} = [8](3)$$

and therefore, finally, at the instant the beetle passes through the point $(3, 2)$,

$$\dot{y} = 8.$$

PROBLEM 5. What is $\dfrac{dy}{dx}$ at the point $(3, 2)$ on the curve $z = f(x, y) = 6$?

ANSWER: $\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}} = \dfrac{8}{3}.$

Suppose that the function f with domain $[0, 6]$ is defined by

$$y = f(x) = x(6 - x),\ 0 \leq x \leq 6.$$

Suppose that $R(x)$ is a rectangle with two sides on the coordinate axes and with a vertex at the origin $(0, 0)$ and with the vertex opposite the origin at the point $(x, y) = (x, f(x)),\ 0 \leq x \leq 6$. Let A denote the area of the rectangle as a function of x, so $A(x)$ denotes the area of $R(x)$ for $0 \leq x \leq 6$. Then

$$A(x) = xy = xf(x) = x^2(6 - x) = 6x^2 - x^3,\ 0 \leq x \leq 6.$$

PROBLEM 6. What is $A'(x)$ for $0 < x < 6$?

ANSWER: $A'(x) = 12x - 3x^2 = 3x(4 - x)$.

PROBLEM 7. What are the critical points of A on the interval $0 \leq x \leq 6$?

ANSWER: Since the critical points are any points where either $A' = 0$ or where A' is undefined, and as A' is defined for all x with $0 \leq x \leq 6$, with

$$A'(x) = 3x(4 - x),$$

which is obviously zero if and only if x is zero or four,
it follows that the critical points of A on the interval $0 \leq x \leq 6$ are $x = 0$ and $x = 4$.

PROBLEM 8. What is $A''(x)$ on the interval $0 < x < 6$?

ANSWER: As $A'(x) = 12x - 3x^2$, it follows that

$$A''(x) = 12 - 6x = 6[2 - x],\ 0 < x < 6.$$

PROBLEM 9. What are the inflection points of A on the interval $0 < x < 6$?

ANSWER: The inflection points are the points where A'' changes sign, and as A'' is defined for all x with $0 < x < 6$, it follows that inflection points are just the points where

$$A'' = 0.$$

Since $A''(x) = 6(2-x)$, it follows that there is only one inflection point which happens at $x = 2$.

PROBLEM 10. What is the value of x for which the area $A(x)$ is maximum for $0 \leq x \leq 6$?

ANSWER: The maximum (as well as the minimum) of A for $0 \leq x \leq 6$ must occur at a critical point or at one of the boundary points,either $x = 0$ or $x = 6$. But at $A(0) = 0 = A(6)$, so the maximum must occur at $x = 4$. In fact, $A(4) = (4^2)(6 - 4) = (16)(2) = 32$, so the maximum the area can be is 32, achieved by taking $x = 4$.

Calclate the following integrals and derivatives.

ANSWERS:

PROBLEM 11. $\displaystyle\int_0^4 \sqrt{x}dx = \int_0^4 x^{1/2}dx = \left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{2}{3}\left[x\sqrt{x}\right]_0^4 = \frac{2}{3}[8 - 0] = \frac{16}{3}$

PROBLEM 12. $\displaystyle\frac{d}{dx}\int_{-2}^{x}\left[\sqrt{t^4 + t^2 + 2}\right]dt = \sqrt{x^4 + x^2 + 2}$