MATH-1150 (DUPRÉ) SPRING 2013 TEST 4 ANSWERS

THE USUAL DIRECTIONS (AS ON TEST 3) APPLY.

Suppose that the function $f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ is given by

$$f(x, y) = x^2y - xy - xy^2 = xy(x - y - 1).$$

We can easily see that

$$f(4, 2) = 8.$$

Therefore the point $(4, 2)$ is on the curve with equation $f(x, y) = 8$ which is a curve in the $(x, y)-$plane.

PROBLEM 2. Suppose that x and y are both functions of time, t, that $x = x(t)$, and $y = y(t)$, and the function $z = z(t)$ of time is given by $z = f(x, y)$, so that

$$z = x^2y - xy - xy^2 = z(t) = f(x(t), y(t)) = [x(t)]^2y(t) - x(t)y(t) - x(t)[y(t)]^2, t \in \mathbb{R}.$$

Use the chain rule to express $\dot{z}$ in terms of $x,\ y,\ \dot{x},\ \dot{y}$.

ANSWERS: $\dot{z} = [2xy - y - y^2]\dot{x} + [x^2 - x - 2xy]\dot{y} = y[2x - y - 1]\dot{x} + x[x - 2y - 1]\dot{y}$

PROBLEM 3. Suppose that a beetle is crawling along the graph of the curve

$$z = f(x, y) = 8$$

so that the rate of increase of x for the beetle is 6 units per second at the instant the beetle passes through the point $(4, 2)$. What is the rate of increase of z for the beetle at the instant the beetle passes through the point $(4, 2)$?

ANSWERS: $\dot{z} = 0$

PROBLEM 4. Suppose that a beetle is crawling along the graph of the curve

$$z = f(x, y) = 8$$

so that the rate of increase of x for the beetle is 6 units per second at the instant the beetle passes through the point $(4, 2)$. What is the rate of increase of y for the beetle at the instant the beetle passes through the point $(4, 2)$?

ANSWERS: Since $0 = \dot{z} = [2xy - y - y^2]\dot{x} + [x^2 - x - 2xy]\dot{y} = y[2x - y - 1]\dot{x} + x[x - 2y - 1]\dot{y}$ for the beetle, it follows that at the point $(4, 2)$,

$$10\dot{x} - 4\dot{y} = 0.$$

Therefore, $\dot{y} = [(10)/4]\dot{x} = [5/2][6] = (5)(3) = 15.$

PROBLEM 5. What is $\dfrac{dy}{dx}$ at the point $(4,2)$ on the curve $z = f(x,y) = 8$?

ANSWERS: The beetle knows. At the point $(4,2)$ moving along the curve $z = 8$ with $\dot{x} = 6$ the rate of change of y is $\dot{y} = 15$, so

$$\frac{dy}{dx} = \frac{15}{6} = \frac{5}{2} = 2.5.$$

Suppose that the function f with domain $[-1,12]$ is defined by

$$y = f(x) = x(12-x), \ -1 \leq x \leq 12.$$

Suppose that

$$g(x) = xy = xf(x) = x^2(12-x) = 12x^2 - x^3, \ -1 \leq x \leq 12.$$

PROBLEM 6. What is $g'(x)$ for $-1 < x < 12$?

ANSWERS: $g'(x) = 24x - 3x^2 = 3x(8-x)$.

PROBLEM 7. What are the critical points of g on the interval $-1 < x < 12$?

ANSWERS: The critical points are the points where g' has value zero. Since $g'(x) = 3x(8-x)$, it follows that g' will be zero only for $x = 0, 8$, so these are the critical points of g.

PROBLEM 8. What is $g''(x)$ on the interval $-1 < x < 12$?

ANSWERS: $g''(x) = [g'(x)]' = [24x - 3x^2]' = 24 - 6x = 6(4-x)$.

PROBLEM 9. What are the inflection points of g on the interval $-1 < x < 12$?

ANSWERS: The inflection points of g are the points where g'' is zero, so as $g''(x) = 6(4-x)$, it follows that $x = 4$ is the only inflection point of g.

PROBLEM 10. What is the value of x for which $g(x)$ is maximum for $-1 \leq x \leq 12$?

ANSWERS: The value of x giving the maximum value of g on the interval $-1 \leq x \leq 12$, is either a boundary point or critical point of g in the interval, so

$$\text{the only possibilities are } x = \ -1, \ 0, \ 8, \ 12.$$

Now using $g(x) = x^2(12-x)$, we find

$$g(-1) = 13, \ g(0) = 0, \ g(8) = 256, \ g(12) = 0,$$

and therefore clearly $x = 8$ gives the maximum value of g on the interval $-1 \leq x \leq 12$.

Calclate the following integrals and derivatives.

ANSWERS:

PROBLEM 11. $\displaystyle\int_0^8 x^{2/3}\,dx = \left[\frac{x^{5/3}}{5/3}\right]_0^8 = \frac{3}{5}\left[x^{5/3}\right]_0^8 = \frac{3}{5}8^{5/3} = \frac{3}{5}\left[8^{1/3}\right]^5 = \frac{3}{5}2^5 = \frac{96}{5}.$

PROBLEM 12. $\displaystyle\frac{d}{dx}\int_{-2}^{x} e^{\sqrt{t^4+2}}\,dt = e^{\sqrt{x^4+2}}$