## MATH-1230 (DUPRÉ) FALL 2010 TEST 6 TAKE HOME

**FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF THIS SHEET.**

**SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME.**

**THIRD: WRITE YOUR FALL 2010 MATH-1230 LAB DAY DIRECTLY UNDERNEATH YOU FIRST NAME.**

**DIRECTIONS: WRITE YOUR FINAL ANSWERS IN THE SPACE PROVIDED ON THE TEST SHEET. WRITE YOUR FULL SOLUTION TO EACH PROBLEM ON A SHEET OF PLAIN WHITE PAPER SHOWING ALL YOUR WORK WITH EACH SOLUTION ON A SEPARATE SHEET OF PAPER USING ONE SIDE ONLY. DO NOT WRITE ON THE BACK OF ANY SHEET RURNED IN. FOLLOW STEPS ABOVE FOR EACH SHEET TURNED IN FOR IDENTIFICATION PURPOSES. EACH PROBLEM IS WORTH 5 POINTS. THERE ARE 20 PROBLEMS.**

Suppose that $X_1, X_2, X_3, ..., X_n, ...$ is a sequence of uncorrelated random variables all having mean $\mu$ and standard deviation $\sigma$. Let

$$T_n = \sum_{k=1}^{n} X_k, \ \ n = 1, 2, 3, ...$$

and let

$$\bar{X}_n = \frac{1}{n} T_n, \ \ n = 1, 2, 3, ...,$$

so we are assuming

$$Cov(X_i, X_j) = \sigma^2, \ \ i = j, \textbf{and} \ Cov(X_i, X_j) = 0, \ \ i \neq j.$$

**1.** What is $Cov(X_3, \bar{X}_2)$?

**2.** What is $Cov(X_2, \bar{X}_3)$?

**3.** What is $Cov(T_2, \bar{X}_3)$?

**4.** What is $Cov(\bar{X}_2, \bar{X}_3)$?

**5.** What is $Cov(\bar{X}_4, \bar{X}_7)$?

**6.** What is $Cov(\bar{X}_4, X_2 - \bar{X}_7)$?

**7.** What is $Cov(X_7 - \bar{X}_6, X_5 - \bar{X}_4)$?

Suppose that $X$ is an unknown with probability density function (pdf) satisfying

$$f_X(x) = \frac{3}{4}(1 - x^2), \ -1 \le x \le 1.$$

**8.** What is $P(X \le -2)$?

**9.** What is $P(X \le 0)$?

**10.** What is $P(X \ge .4)$?

**11.** What is $P(-.2 \le X \le .3)$?

**12.** What is $E(X)$?

**13.** What is $E(X^2)$?

**14.** What is $\sigma_X^2$, the variance of $X$?

**15.** If $T$ is the total of 5 random observations of $X$, then what is $E(T)$?

**16.** If $T$ is the total of 5 independent random observations of $X$, then what is $\sigma_T^2$, the variance of $T$?

Suppose that $A$ is a statement with $p = P(A) = E(I_A)$, where $I_A$ is the indicator unknown of $A$. Let $T_n$ be the total of $n$ independent observations of $I_A$.

**17.** What is $m_{I_A}(t)$, the moment generating function of $I_A$ in terms of $p, t$?

**18.** What is the formula or expression for the moment generating function $m_{T_n}(t)$ for $T_n$ in terms of $t, n, p$?

**19.** What is the mean and variance of $T_n$ in terms of $n$ and $p$?

**20.** What is $P(T_n = k)$ in terms of $n, k, p$, and what is $P(T_n = 4)$ for the case where $n = 10$ and $p = .3$?