MATH-1XXX (DUPRÉ) FALL 2013 TEST 2 ANSWER DETAILS

DATE: WEDNESDAY 2 OCTOBER 2013

1. PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

2. PRINT YOUR FIRST NAME IN LARGE CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

3. WRITE YOUR MATH COURSE NUMBER AND SECTION NUMBER DIRECTLY UNDERNEATH YOUR FIRST NAME ON EACH SHEET TURNED IN.

In each of the following PROBLEMS below, CIRCLE the BOLDFACE LETTER indicating the CORRECT ANSWER.

Suppose that X **and** Y **are unknowns with** $\mu_X = 70,\ \sigma_X = 10,\ \mu_Y = 50,$ **and** $\sigma_Y = 5.$ **Further suppose that the correlation coefficient giving the correlation of** X **with** Y **is** $\rho = .8.$ **Answer the following questions using this information.**

4. The VARIANCE of X is =

A. $\sqrt{10}$
B. 10
C. 100
D. 4900
E. None of the above

CORRECT ANSWER CHOICE: C

The relationship between the variance of X, denoted $Var(X)$, and the standard deviation of X, denoted $SD(X) = \sigma_X$, is that the standard deviation is the square root of the variance and therefore the variance is the square of the standard deviation:

$$SD(X) = \sigma_X = \sqrt{Var(X)} \text{ and } Var(X) = \sigma_X^2 = [SD(X)]^2.$$

Here, the standard deviation of X is given to be $\sigma_X = 10$, so the variance of X is $(10)^2 = 100$. Therefore the correct answer choice is **C.**

5. If a particular score for X is $x = 95$, then the equivalent standard score is $z_x =$

A. 1.5

B. 2

C. 2.5

D. 3

E. None of the above

CORRECT ANSWER CHOICE: C

If U is any unknown with mean μ_U and standard deviation σ_U,, then its **standardization**, denoted Z_U is given by

$$Z_U = \frac{U - \mu_U}{\sigma_U}.$$

Therefore, the standardization of X is

$$Z_X = \frac{X - \mu_X}{\sigma_X} = \frac{X - 70}{10}.$$

In particular, a raw score of $x = 95$ is equivalent to the standard score z_x, where

$$z_x = \frac{95 - 70}{10} = \frac{25}{10} = 2.5.$$

Therefore the correct answer choice is **C.**

6. If a particular standard score for Y is $z_y = 2.7$, then the actual equivalent raw score for Y is $y =$

A. 63.5

B. 73.5

C. 83.5

D. 97

E. None of the above

CORRECT ANSWER CHOICE: A

If U is any unknown with mean μ_U and standard deviation σ_U,, then its **standardization**, denoted Z_U is given by

$$Z_U = \frac{U - \mu_U}{\sigma_U}.$$

Therefore, the unknown is recovered from its standardization using the formula

$$U = \mu_U + Z_U \cdot \sigma_U.$$

Applying this to Y, we have

$$Y = \mu_Y + Z_Y \cdot \sigma_Y = 50 + Z_Y \cdot 5 = 50 + 5Z_Y.$$

In particular, the raw score equivalent to the standard score $z_y = 2.7$ is the raw score y, where

$$y = 50 + 5(2.7) = 50 + 13.5 = 63.5.$$

Therefore the correct answer choice is **A.**

7. Given that a particular score for X is equivalent to the standard score $z_x = 2$, then, using the correlation of X with Y, the corresponding standard score that should be guessed for Y is the standard score

$z_y = E(Z_Y|Z_X = 2) =$

A. 1.4

B. 1.6

C. 1.8

D. 2.0

E. None of the above

CORRECT ANSWER CHOICE: B

In general, we know that using the correlation of X with Y that

$$E(Z_Y|Z_X = z) = \rho \cdot z.$$

Therefore,

$$z_y = E(Z_Y|Z_X = 2) = \rho \cdot 2 = (.8)(2) = 1.6.$$

The correct answer choice is therefore **B.**

8. Given a particular score $x = 90$ for X, then using the correlation of X with Y, the corresponding score that should be guessed for Y is

$y = E(Y|X = 90) =$

A. 58

B. 60

C. 86

D. 90

E. None of the above

CORRECT ANSWER CHOICE: A

A raw score of $x = 90$ is equivalent to the standard score of $z_x = (90 - 70)/(10) = 2$, so just as in the preceding problem, the standard score we should guess for Y is simply $z_y = 1.6$ which is equivalent to the raw score of

$$y = 50 + 5(1.6) = 58.$$

The correct answer choice is therefore **A.**

9. Given that we DO NOT USE the correlation of X with Y to guess a value for Y, but simply guess $E(Y) = 50$, without looking to see the value of X, then our expected squared error is

$E(\text{error}^2) =$

A. 50
B. 25
C. 10
D. $\sqrt{5}$
E. None of the above

CORRECT ANSWER CHOICE: B

In case we guess Y without using any knowledge of X, we would guess the value $\mu_Y = 50$, and our expected squared error is

$$E(\text{error}^2) = E[(Y - \mu_Y)^2] = Var(Y) = \sigma_Y^2 = 5^2 = 25.$$

The correct answer choice is therefore **B.**

10. Given that we DO USE the correlation of X with Y to guess a value of Y from an observed value of X using linear regression properly, then our expected squared error is

$E(\text{error}^2 | X \text{ value}) =$

A. $(1 - \rho^2)\sigma_Y^2$
B. $(1 - \rho)\sigma_Y$
C. $\rho^2\sigma_Y^2$
D. $\rho\sigma_X\sigma_Y$
E. None of the above

CORRECT ANSWER CHOICE: A

The linear regression equation gives the minimum expected squared error possible given by the equation

$$E(\text{error}^2 | X \text{ value}) = E[(Y - [\alpha + \beta \cdot X])^2 | X \text{ value}) = (1 - \rho^2)\sigma_Y^2,$$

where

$$\beta = \rho\frac{\sigma_Y}{\sigma_X} \text{ and } \alpha = \mu_Y - \beta \cdot \mu_X.$$

Therefore the correct answer choice is **A.**

11. The COVARIANCE of X with Y is equal to

A. $(1-\rho^2)\sigma_Y^2$

B. $(1-\rho)\sigma_Y$

C. $\rho^2\sigma_Y^2$

D. $\rho\sigma_X\sigma_Y$

E. None of the above

CORRECT ANSWER CHOICE: D

The covariance of X with Y, denoted $Cov(X,Y)$, is

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - \mu_X \cdot \mu_Y = \rho\sigma_X\sigma_Y.$$

The correct answer choice is therefore **D.**

12. The VARIANCE of $X - Y$ is equal to

A. 5

B. 25

C. 75

D. 125

E. None of the above

CORRECT ANSWER CHOICE: E

The variance of a sum or difference is given by

$$Var(X \pm Y) = Var(X) + Var(Y) \pm 2 \cdot Cov(X,Y),$$

which, using $Cov(X,Y) = \rho\sigma_X\sigma_Y$, is equivalent to

$$Var(X \pm Y) = \sigma_X^2 + \sigma_Y^2 \pm 2\rho\sigma_X\sigma_Y,$$

so with the numbers here, we have

$$Var(X - Y) = 100 + 25 - 2(.8)(10)(5) = 100 + 25 - 80 = 45.$$

The correct answer choice is therefore **E.**

Suppose that A and B are events with $P(A) = .6$, $P(B) = .4$, and $P(A|B) = .5$. Answer the following questions using this information.

13. $P(A \text{ and } B) =$

A. 0.24
B. 1
C. .3
D. .2
E. None of the above

CORRECT ANSWER CHOICE: D

The formula for the multiplication rule in probability is

$$P(A \,\&\, B) = P(A|B) \cdot P(B),$$

so here we have

$$P(A \,\&\, B) = (.5)(.4) = .2,$$

so the correct answer choice is **D.**

14. $P(A \text{ or } B) =$

A. 1
B. .8
C. .6
D. .4
E. None of the above

CORRECT ANSWER CHOICE: B

In general,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \,\&\, B),$$

so here we have

$$P(A \text{ or } B) = (.6) + (.4) - (.2) = .8,$$

so the correct answer choice is **B.**

15. $P(A \text{ but not } B) =$

A. 0.2

B. 0.4

C. 0.6

D. 0.8

E. None of the above

CORRECT ANSWER CHOICE: B

In general,

$$P(A \text{ but not } B) = P(A) - P(A \ \& \ B),$$

so here,

$$P(A \text{ but not } B) = .6 - .2 = .4,$$

making the correct answer choice **B.**

16. The events A and B are [**CIRCLE AT LEAST ONE**]

A. MUTUALLY INDEPENDENT

B. MUTUALLY EXCLUSIVE

C. NEITHER INDEPENDENT NOR EXCLUSIVE

D. BOTH INDEPENDENT AND EXCLUSIVE

E. None of the above

CORRECT ANSWER CHOICE: C

Since $P(A \ \& \ B) = .2$, it follows that the two events are NOT mutually exclusive, and as $P(A|B) \neq P(A)$, it follows that the two events are NOT mutually independent. Moreover, if the events were both independent and exclusive, then we would have $P(A) \cdot P(B) = P(A \ \& \ B) = 0$, so at least one of the events would be the impossible event, which is clearly not the case here. Therefore the correct answer choice is **C.**

Suppose that we have twenty cards from a standard deck of cards so as to have 5 of each suit. Suppose that we deal out four cards from this deck of twenty cards. Answer the following questions using this information.

17. What is the probability that all four cards are of the same suit?

A. $C(5,4)/C(20,4)$
B. $C(5,4)/P(20,4)$
C. $C(5,4)/C(20,5)$
D. $C(20,4)/C(20,5)$
E. None of the above

CORRECT ANSWER CHOICE: E

To choose four cards from this deck so as to have all the same suit, you must first decide which of the four suits to use and then pick four of the five cards of that suit. The number of ways to pick one of the four suits is simply $C(4,1) = 4$, and the number of ways to pick four of the five cards of that suit is $C(5,4) = C(5,1) = 5$. Therefore the number of ways to choose four cards so as to have all the same suit is

$$C(4,1) \cdot C(5,4) = 4 \cdot 5 = 20.$$

The number of ways to choose four cards from a deck of 20 cards is simply $C(20,4) = 4845$. It follows that the probability of dealing four cards of the same suit is

$$P(\text{all four cards of the same suit}) = \frac{C(4,1) \cdot C(5,4)}{C(20,4)} = \frac{20}{4845} = 0.004127967.$$

This means the correct answer choice is **E.**

18. What is the probability that there are three spades and one heart?

A. $C(5,3)C(5,1)/C(5,4)$
B. $C(5,3)C(5,1)/C(20,4)$
C. $P(5,3)P(5,4)/P(5,5)$
D. $C(5,2)C(5,3)/C(20,4)$
E. None of the above

CORRECT ANSWER CHOICE: B

To choose four cards so as to get three spades and one heart, you can first choose three of the five spades, which can be done in $C(5,3) = 10$ ways and then choose one of the five hearts which can be done in $C(5,1) = 5$ ways, so there are

$$C(5,3) \cdot C(5,1) = (10)(5) = 50$$

possible four card hands having three spades and one heart. As there are $C20,5) = 4845$ possible four card hands, the probability of being dealt such a hand is

$$P(\text{three spades and one heart}) = \frac{C5,3) \cdot C(5,1)}{C(20,4)} = \frac{50}{4845} = 0.0103199174.$$

The correct answer choice is therefore **B.**