

MATH-1230 (DUPRÉ) SPRING 2018 TEST 1 ANSWERS

1. PRINT YOUR LAST NAME IN THE UPPER RIGHT CORNER IN LARGE CAPITAL LETTERS.

2. PRINT YOUR FIRST NAME UNDERNEATH YOUR LAST NAME IN THE UPPER RIGHT CORNER IN CAPITAL LETTERS.

3. PRINT YOUR LAB DAY AND LAB START TIME UNDERNEATH YOUR FIRST NAME IN THE UPPER RIGHT CORNER.

4. WRITE YOUR SPRING 2018 MATH-1230 COURSE SECTION NUMBER UNDERNEATH YOUR LAB DAY IN THE UPPER RIGHT CORNER.

The remaining problems all use the information that follows. Suppose that Joe has an aquarium tank containing  $N = 10$  fish. For any fish selected, we let  $X$  denote the LENGTH of the fish in millimeters and  $W$  the WEIGHT of the fish in milligrams. Joe tells his friend Sam that the fish in his tank have an average length of 30 millimeters with a standard deviation of 2 millimeters, and that the average weight of the fish is 20 milligrams with a standard deviation of 4 milligrams. Joe also tells Sam that the correlation coefficient of length and weight is 0.8. Suppose that Joe has selected two fish from the aquarium, one after another, without letting Sam see which fish he selected, and placed the fish in a smaller holding tank for later observation. Let the unknown  $W_1$  be the weight of the first fish Joe has selected from the aquarium, which Sam does not know, and henceforth called "the first fish". Let  $W_2$  be the weight of the second fish Joe selected and henceforth known as "the second fish". Likewise, let the unknown  $X_1$  be the length of the first fish, and likewise, let  $X_2$  be the length of the second fish.

5. Without knowing the LENGTH of the either fish, what should Sam expect to be its WEIGHT?

ANSWERS: 20

6. Without knowing the LENGTH of either fish, what should Sam expect to be the AVERAGE WEIGHT of the two fish that Joe selected from the aquarium?

ANSWERS: 20

7. Without knowing the length of either fish, what should Sam expect for his squared error in his expected WEIGHT of the second fish?

ANSWERS:  $4^2 = 16$

8. Without knowing the LENGTH of either fish, what should Sam expect for his squared error in his expected AVERAGE WEIGHT of the two fish that Joe selected from the aquarium?

ANSWERS:  $\frac{10-2}{10-1} \cdot \frac{4^2}{2} = \frac{64}{9} = 7.111\dots = 7.\bar{1}$ , or about 7.11, to 2 decimal place accuracy.

9. If the first fish is 36 millimeters long, what is its STANDARDIZED LENGTH score?

ANSWERS:  $\frac{36-30}{2} = 3$

10. If the first fish has a STANDARDIZED WEIGHT score of -1.5, then what is its RAW WEIGHT score, that is its weight in milligrams?

ANSWERS:  $20 + (4)(-1.5) = 14$

11. If Sam is told that the second fish is 30 millimeters LONG, then what should Sam expect for the WEIGHT of the second fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWERS: He should expect an average length fish to be of average weight since  $\rho \cdot 0 = 0$ , so expect 20.

12. If Sam is told that the first fish is 36 millimeters LONG, then what should Sam expect for the WEIGHT of the first fish, using SIMPLE LINEAR REGRESSION in view of the given correlation of length and weight?

ANSWERS:  $20 + (4)(.8)\frac{36-30}{2} = 29.6$

13. In the previous two cases where Sam knew the lengths of the two fish, what should Sam expect is his squared error in his expected value for the WEIGHT of the fish when using SIMPLE LINEAR REGRESSION and the correlation of length and weight to determine what WEIGHT to expect by using the LENGTH?

ANSWERS:  $(4^2)(1 - (.8^2)) = 5.76$

14. What is the COVARIANCE of  $X$  with  $W$ ?

ANSWERS:  $(.8)(2)(4) = 6.4$

15. What is the expected product of  $X$  and  $W$ , that is, what is  $E(XW)$ ?

ANSWERS:  $(30)(20) + (.8)(2)(4) = 606.4$

16. If  $T = X + W$ , then what is the VARIANCE of  $T$ ?

ANSWERS:  $2^2 + 4^2 + 2(.8)(2)(4) = 32.8$

17. If  $D = X - W$ , then what is the VARIANCE of  $D$ ?

ANSWERS:  $2^2 + 4^2 - 2(.8)(2)(4) = 7.2$

18. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, insuring that the second fish selected could also be any one of the 10 fish in the aquarium, then what is the correlation of  $X_1$  with  $X_2$ ?

ANSWERS: 0

19. If the first fish is put back in the aquarium before the second fish is selected and the second fish is selected without knowing which fish was the first fish selected, and if  $T = X_1 + X_2$ , then what is the VARIANCE of  $T$ ?

ANSWERS:  $2^2 + 2^2 = 8$

20. If Bill (where this guy comes from or what he knows we care NOT) guesses that the first fish is 33 millimeters LONG, then what should Sam expect for Bill's squared error in this guess for the LENGTH, assuming that Sam does not know the lengths of these fish either?

ANSWERS:  $(33 - 30)^2 + 2^2 = 13$