MATH-1230 (DUPRÉ) FALL 2010 TEST 4 ANSWERS

FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF THIS SHEET.

SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME.

THIRD: WRITE YOUR FALL 2010 MATH-1230 LAB DAY DIRECTLY UNDERNEATH YOU FIRST NAME.

My acquarium contains a population consisting of 4 fish: Amy, Jill, Joe, and Sam. Amy is 4 cm long and weighs 2 gm, Jill is 6 cm long and weighs 3 gm, Joe is 7 cm long and weighs 3 gm, and Sam is 8 cm long and weighs 4 gm. Let X be length and let Y be weight for this population.

NOTE: The answers are most efficiently calculated with the calculator using the statistical 2-Var Stats and the LinReg(a+bx) in the calculator. Using two lists, all the answers can be read off the readout.

1. What is the mean of X?

ANSWER

The values of X are $4,\ 6,\ 7,\ 8$ all equally likely, so the mean of X is simply $(4+6+7+8)/4 = 25/4 = 6.25$.

FINAL ANSWER: 6.25

2. What is the standard deviation of X?

ANSWER

The variance of X can be easily calculated by the formula $Var(X) = E(X^2) - \mu_X^2$. Since the variance of X is the same as that of $X - c$ where c is any constant, we can take $c = 6$ for convenience, and this replaces the values of X by the values of $X - 6$ which are $-2,\ 0,\ 1,\ 2$, whose average is obviously just .25 and whose squares are therefore $4,\ 0,\ 1,\ 4$. Thus the average of the squares is $9/4$ or 2.25, and therefore

$$Var(X) = Var(X-6) = \frac{9}{4} - \frac{1}{4}\cdot\frac{1}{4} = \frac{35}{16}.$$

The standard deviation of X is then

$$\sigma_X = \sqrt{Var(X)} = \sqrt{35/16} = 1.479019946,$$

which to three significant digits is 1.479, since a leading one does not count as a significant digit.

FINAL ANSWER: 1.479

3. What is the mean of Y?

ANSWER

Since the values of Y are $2,\ 3,\ 4$ with $P(Y=2) = 1/4 = P(Y=4)$ and $P(Y=3) = 1/2$, it is obvious that $E(Y) = 3$. In detail, $\mu_Y = (2+3+3+4)/4 = 12/4 = 3$.

FINAL ANSWER: 3

4. What is the standard deviation of Y?

ANSWER

Since the deviation of Y has the values the values $-1,\ 0,\ 1,$ with -1 and 1 each having probability $1/4$ and with 0 having probability $1/2$, it follows that the squared deviation has values 0 or 1 each with probability $1/2$ and therefore the variance which is the mean of the squared deviation is simply $1/2$. That is,

$$Var(Y) = \frac{1}{2} = .5,$$

so the standard deviation of Y is simply

$$\sigma_Y = \sqrt{.5} = .7071067812,$$

or to three significant digits, $\sigma_Y = .707$.

FINAL ANSWER: .707

5. What is the correlation coefficient of X and Y?

ANSWER

The correlation is given by

$$\rho = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{E(XY) - \mu_X \cdot \mu_Y}{\sqrt{Var(X) \cdot Var(Y)}} = \frac{E(XY) - 18.75}{\sqrt{(35/16)(1/2)}}.$$

Now the values of XY are $8,\ 18,\ 21,\ 32,$ all equally likely, so $E(XY) = 79/4 = 19.75$. It follows that the covariance of X with Y is just

$$Cov(X,Y) = E(XY) - \mu_X \cdot \mu_Y = 19.75 - 18.75 = 1.$$

This means that

$$\rho = \sqrt{(16/35)(2)} = 4 \cdot \sqrt{2/35} = .9561828875.$$

FINAL ANSWER: .9561828875

6. If Amy gains one cm in length how much should we guess her weight to increase?

ANSWER

We use β_1 to denote the slope of the regression line. In general then, if we see a unit increase in the value of X we would increase our previous guess for Y by β_1. We only need to calculate β_1 and it is given by the formula

$$\beta_1 = \rho \cdot \frac{\sigma_Y}{\sigma_X} = \frac{\rho \cdot \sigma_X \cdot \sigma_Y}{\sigma_X^2} = \frac{Cov(X,Y)}{Var(X)} = \frac{1}{35/16} = \frac{16}{35} = .4571428571,$$

or simply .457 to three significant digits.

FINAL ANSWER: .457

NOTE: There are actually two answers acceptable here. If you use the regression equation, you get

$$\beta_0 = \mu_Y - \beta_1 \cdot \mu_X = 3 - \frac{16}{35} \cdot \frac{25}{4} = 3 - \frac{20}{7} = \frac{21}{7} - \frac{20}{7} = \frac{1}{7},$$

so finally,

$$\beta_0 = \frac{1}{7} = .1428571429,$$

and the regression equation is

$$W = \frac{1}{7} + \frac{16}{35}X.$$

This means if Amy increases to 5 cm in length, when we use the regression equation directly to guess the weight of a fish of this length we would guess

$$w = \frac{1}{7} + \frac{16}{35} \cdot 5 = \frac{1}{7} + \frac{16}{7} = \frac{17}{7} = 2.428571429,$$

and this would mean an increase of .428571429 gm for Amy instead of .457. But, guessing Amy's new weight to be given by the regression equation overlooks the fact that we know her original weight and length. Indeed, notice that the regression equation does not give Amy's weight for her original lenght of 4 cm, since the regression equation says to guess the weight of a 4 cm fish to be 69/35 gm, so Amy is maybe 1/35 gm overweight according to the regression equation. This might make us think that Amy is a type of fish that is heavier for a given length than average and so maybe we should even guess more than a .457gm increase for Amy in this situation. You can see that there are problems here that can be argued about.