# MATH-1230 (DUPRÉ) FALL 2010 TEST 5 ANSWERS

**FIRST: PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF THIS SHEET.**

**SECOND: PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME.**

**THIRD: WRITE YOUR FALL 2010 MATH-1230 LAB DAY DIRECTLY UNDERNEATH YOU FIRST NAME.**

**Suppose that $X$ is a random variable with mean $\mu_X = 80$ and standard deviation $\sigma_X = 10$. Suppose that $Z$ is the standardization of $X$. Suppose that $X_1, X_2, X_3, ..., X_n$ is a sequence of $n$ observations of $X$ constituting a sample of size $n$. We use IRS for independent random sample and SRS for simple random sample (drawing without replacement from a population of size $N$.) We denote by $T$ the sample total and by $\bar{X}$ the sample mean. Thus $T$ and $\bar{X}$ are new unknowns given by**

$$T = X_1 + X_2 + X_3 + ... + X_n$$

**and**

$$\bar{X} = \frac{1}{n}T.$$

**1.** If $X = 97$ then what is the value of $Z$?

**ANSWER:**
In general,

$$Z_X = \frac{X - \mu_X}{\sigma_X}, \; X = \mu_X + \sigma_X \cdot Z_X$$

are the equations relating $X$ and its standardization $Z_X$, which get you back and forth between ordinary scores and standard scores. So, as $\mu_X = 80$ and $\sigma_X = 10$, it follows that if $X = 97$, then

$$Z_X = \frac{97 - 80}{10} = 1.7.$$

**2.** If $Z = 1.4$, then what is the value of $X$?

**ANSWER:**
In general,

$$Z_X = \frac{X - \mu_X}{\sigma_X}, \; X = \mu_X + \sigma_X \cdot Z_X$$

are the equations relating $X$ and its standardization $Z_X$, which get you back and forth between ordinary scores and standard scores. So, as $\mu_X = 80$ and $\sigma_X = 10$, it follows that if $Z_X = 1.4$, then

$$X = 80 + (10)(1.4) = 80 + 14 = 94.$$

**3.** What is $E(\bar{X})$?

**ANSWER:**

For any size sample of $X$ using any sampling method we have

$$E(\bar{X}) = \mu_X,$$

therefore in the case at hand,

$$E(\bar{X}) = 80.$$

**4.** If $n = 4$, what is $E(T)$?

**ANSWER:**

For any sample of size $n$ and any sampling method we have

$$E(T_n) = n \cdot \mu_X,$$

so here we have

$$E(T) = 4 \cdot (80) = 320.$$

**5.** Assuming IRS and $n = 16$, what is $\sigma_T$?

**ANSWER:**

In general, for an IRS of size $n$ we a have

$$\sigma_T = \sqrt{n} \cdot \sigma_X,$$

so here we have

$$\sigma_T = \sqrt{16} \cdot (10) = 4 \cdot (10) = 40.$$

**6.** Assuming SRS, $n = 16$, and $N = 17$, what is $\sigma_T$?

**ANSWER:**

In general, for using SRS instead of IRS, standard deviations for $T$ and $\bar{X}$ must have their IRS values multiplied by the SRS correction factor, $c_{SRS}$ given by

$$c_{SRS} = \sqrt{\frac{N-n}{N-1}},$$

so here

$$c_{SRS} = \sqrt{\frac{17-16}{17-1}} = \sqrt{\frac{1}{16}} = \frac{1}{4},$$

and therefore

$$(\sigma_T)_{SRS} = \frac{1}{4} \cdot (\sigma_T)_{IRS} = \frac{1}{4} \cdot (40) = 10.$$

You should notice here that with $N = n+1$, the sample total only leaves out a single observation from the population, hence as the population total is a constant, the standard deviation for the sample total is the same as the standard deviation for the only missing observation which is therefore the standard deviation for $X$ itself, namely $\sigma_X = 10$. See if you can demonstrate this for any $N$ using the rules, that is show:

$$[N = n + 1] \Rightarrow [(\sigma_T)_{SRS} = \sigma_X].$$

Here, $\Rightarrow$ is the mathematical symbol for "implies".

**7.** If $n = 100$, what is the probability that $\bar{X}$ is between 79 and 82?

**ANSWER:**

Since we only know the mean and standard deviation of $X$, and likewise $\bar{X}$, the normal distribution should be used, however, even with some other knowledge of the distribution beyond merely the mean and standard deviation, as $n \geq 30$, we can assume $\bar{X}$ and $T$ are normal by the **Central Limit Theorem**. Therefore,

$$P(79 \leq \bar{X} \leq 82) = P(79 < \bar{X} < 82) = \mathit{normalcdf}(79, 82, 80, 10/\sqrt{100})$$
$$= \mathit{normalcdf}(79, 82, 80, 1) = \mathit{normalcdf}(-1, 2, 0, 1) = .8185946784.$$

**8.** If the moment generating function of $Y$ is

$$m_Y(t) = 1 + (1/2)t + (1/3)t^2 + (1/4)t^3 + ...,$$

then what is $E(Y^2)$?

**ANSWER:**

In general, if the moment generating function is given as a power series

$$m_Y(t) = \sum_{n=0}^{\infty} c_n t^n,$$

then

$$E(Y^n) = m_Y^{(n)}(0) = (n!)c_n, \ n = 0, 1, 2, 3, ...,$$

so here

$$E(Y^2) = (2!) \cdot \frac{1}{3} = \frac{2}{3}.$$

Likewise,

$$E(Y^3) = (3!) \cdot \frac{1}{4} = \frac{6}{4} = 1.5,$$

and so on.