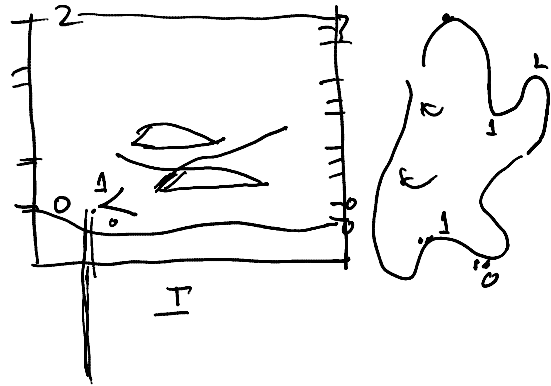# "Broken fibrations of 4-manifolds"
Prof. Rob Kirby.

Morse theory — $X^4 \longrightarrow \mathbb{R}$ critical pts

Cerf diagram graphic

$$X^4 \xrightarrow{f} S^2, \; \beta^2, \; S^1 \times \underline{I}, \; \underline{I} \times \underline{I}$$

Thm Existence : f is homotopic to a broken fibration

     Uniqueness : $\exists$ moves taking f_0 to f_1 Lekili

     (J. Williams S^2)   Gay+K   + Sacki
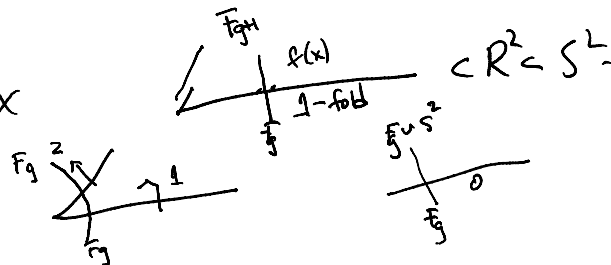
Singularity theory : f is homotopic to a generic map with
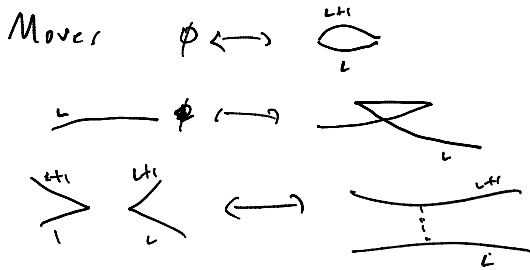
   1) Df has rank 2 at $x \in X$

or 2) Df has rank 1 at $x \in X$

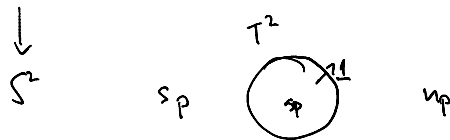   (1-manifold)

     smooth in X

$\subset \mathbb{R}^2 \subset S^2$
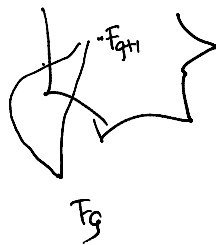
Broken fibration has no definite folds $^{LH}_{=}$

Gay K $\exists$ for $N^n \longrightarrow S^2, B^2, S^1 \times I$
$+$! $\quad$ '' $\quad$ '' $\quad$ ''

Moves $\quad \phi \longleftrightarrow$

$\longrightarrow \phi \longleftrightarrow$

 $\longleftrightarrow$

$S^4 \qquad S^2 \qquad\qquad T^2$

$\downarrow \qquad\qquad T^2$

$S^2 \qquad s_p \quad$ $\quad n_p$

$\pi^4(S^2)$

$X^4 \longrightarrow S^2$

If $\partial X \neq \phi$
Want $\partial X \longrightarrow \partial$ surface
Morse fcn no definite folds

 B^2

J Williams

X^4

The indefinite folds can be chosen connected
i.e. a circle

 many cusps

F_{g+1}

F_g

T Peruta