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For each $j = 1, 2, \ldots, N$, choose $-1 < q_j = \frac{N-2j+1}{N} < 1$ and define

$$f_{j,N}(x) := |x - \frac{N-2j+1}{N}|$$

having the interval $[\frac{N-2j}{N}, \frac{N-2j+2}{N}]$ as a compact support, then the evenly-spaced Saw-Tooth function

$$\rho(x) = min_j|x - q_j| = \sum_{j=0}^{N} f_{j,N}(x)$$

proves that the inequality in Lemma 2, is sharp! That is, for all N

$$\int_{-1}^{1} \rho(x)dx = \frac{1}{N}. \qquad \square$$