The determinant evaluation

$$\det\left[\binom{i+j+a+b}{i+a}\binom{2n-i-j-a-b}{n-i-a}_{0\le i,j\le m}\right]=$$

$$\frac{(a+b)!(2n+1)!^{m+1}(2n-m)!!m!!(m+a+b)!!(2n-m-a-b)!!a!!b!!(n-m-a-1)!!(n-m-b-1)!!}{a!b!(2n+1)!!(n-a)!!(n-b)!!(m+a)!!(m+b)!!(a+b)!!(2n-2m-a-b-1)!!} \tag{Rabbit}$$

is just a special case of an even more general one, namely

$$\det_{0\le i,j\le M}\left(\binom{a_i+j+A}{a_i}\binom{B-j-a_i}{N-a_i}\right)$$
$$=\prod_{i=0}^{M}\frac{(a_i+A)!}{a_i!\,(i+A)!}\frac{(B-M-a_i)!}{(N-a_i)!\,(B-i-N)!}\prod_{0\le i<j\le M}(a_j-a_i)\prod_{1\le i\le j\le M}(A+B+1+i-j). \tag{1}$$

Choosing $M=m$, $a_i=i+a$, $A=b$, $B=2n-b$, $N=n$ in (1) and some simplification gives (*Rabbit*).

The evaluation (1) in turn follows from a determinant lemma of mine (Lemma 2.2 in Manuscripta Math. **69** (1990), 173–202) which seemingly does not receive the attention it deserves:

$$\det_{0\le i,j\le M}\Big((X_i+A_M)\cdots(X_i+A_{j+1})(X_i+B_j)\cdots(X_i+B_1)\Big)$$
$$=\prod_{1\le i<j\le M}(X_i-X_j)\prod_{1\le i\le j\le M}(B_i-A_j).$$

This is seen as follows: Take a few terms out of rows and columns of the determinant in (1) to obtain

$$\prod_{i=0}^{M}\frac{(a_i+A)!}{a_i!}\frac{(B-M-a_i)!}{(N-a_i)!}\prod_{j=0}^{M}\frac{1}{(j+A)!\,(B-j-N)!}$$
$$\times\det\big((a_i+A+1)\cdots(a_i+A+j)\,(B-M-a_i+1)\cdots(B-j-a_i)\big)$$
$$=\prod_{i=0}^{M}\frac{(a_i+A)!}{a_i!}\frac{(B-M-a_i)!}{(N-a_i)!}\prod_{j=0}^{M}\frac{1}{(j+A)!\,(B-j-N)!}$$
$$(-1)^{\binom{M+1}{2}}\det\big((a_i+A+1)\cdots(a_i+A+j)\,(a_i+j-B)\cdots(a_i+M-1-B)\big)$$

for the determinant. Now apply my determinant lemma with $X_i=a_i$, $B_j=A+j$, $A_j=j-1-B$.