MONTHLY PROBLEM

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Theorem:
Suppose $f(x)$ is a positive convex function. Let $a = (a_1 + a_2 + \cdots + a_n)/n$. Then:

$$f(a_1)^{f(a_1)} f(a_2)^{f(a_1)} \dots f(a_n)^{f(a_1)} \geq e^{n[f(a)-1]}. \tag{1}$$

Proof: Recall that $xln(x) - x + 1 = \int_1^x ln(t)dt \geq 0$ and that $f(x)$ is convex. It follows that

$$\frac{1}{n}\sum_{i=1}^{n} f(a_i)ln(f(a_i)) \geq \frac{1}{n}\sum_{1}^{n}[f(a_i)-1] \geq [f(a)-1],$$

where the last inequality follows by Jensen's inequality. Multiplying by n and exponentiating gives the desired inequality. □

Remark: In particular, this proves [P] with $f(x) = \ , (x)$.

Reference:

[**P**] *American Mathematical Monthly - Problem #10549,* **(103) # 8,** 1996.

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