SOLUTION TO PROBLEM #12128

Problem #12128. Proposed by O. Kouba, Syria. Let F_n be the Fibonacci numbers, defined by $F_0 = 0, F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 1$. Find, in terms of n, the number of trailing zeros in the decimal representation of F_n.

Solution by Tewodros Amdeberhan and Victor H. Moll, Tulane University, New Orleans, LA, USA. This is actually a consequence of the result in [1], i.e.

$$(1) \qquad \nu_5(F_n) = \nu_5(n) \qquad \text{and} \qquad \nu_2(F_n) = \begin{cases} 1 & n \equiv 3 \mod 6 \\ \nu_2(4n) & n \equiv 0 \mod 6 \\ 0 & \text{otherwise;} \end{cases}$$

where $\nu_p(\cdots)$ denotes the p-adic valuation function. *The question at hand is simply* the largest power of 10 that divides F_n, denote this by $c(n)$. Observe that 5 divides F_n precisely when it divides n. Also 2 divides F_n when 3 divides n. Therefore $c(n) = 0$ unless 15 divides n. Furthermore, $c(15x) = 1$ as soon as x is odd due to (1). The remaining case is handled by

$$c(30y) = \min\{\nu_2(8y), \nu_5(5y)\}. \qquad \square$$

[1] T. Lengyel, The order of the Fibonacci and Lucas numbers, *The Fibonacci Quarterly,* **33** (1995) 234-239.

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