SOLUTION TO PROBLEM #621
PROPOSED BY J. BEALL AND ET AL

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PROBLEM: [**P**] For a positive integer m, let $\bar{m}$ denote the sum of the digits of m. Find all pairs of positive integers (m,n), with $m<n$, such that $(\bar{m})^2=n$ and $(\bar{n})^2=m$.

SOLUTION: Let $n>m=m_k\dots m_1m_0$, where $0\le m_j\le 9$ are the digits of m. This immediately implies that

$$10^k\le m<n=(m_k+\cdots+m_0)^2\le((k+1)10)^2,$$

that is $10^{k-2}\le(k+1)^2$. Consequently, $0\le k\le 3$.

Hence, $m<n=(m_3+m_2+m_1+m_0)^2\le(4(9))^2=36^2$. This, combined with the fact that m and n are perfect squares show that it suffices to test only the *first thirty six* perfect squares.

An easy check yields $(13^2,16^2)=(169,256)$ as *the only such pair!* $\square$

References:

[**P**] P #621, *The College Mathematics Journal,* **(29) #2,** 1998.

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