SOLUTION TO PROBLEM #634
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Problem #634: Find all real-valued functions f for which there exists a real-valued function g such that $f(x+y)f(x-y) = (x-y)g(x+y)$ for all real numbers x and y.

Solution: Using the bijective transformation $u = x + y$ and $w = x - y$, the condition on f and g reads

$$f(u)f(w) = wg(u). \tag{1}$$

Now, replacing $w = 1$ gives $g(u) = f(1)f(u)$. With this, equation (1) takes the form

$$f(w) = f(1)w \qquad \text{and} \qquad g(u) = f(1)^2 u.$$

Consequently, the only real-valued functions fitting the desired relation are of the type

$$f(x) = \alpha x, \qquad \text{where } \alpha \text{ is } \textit{any} \text{ constant!}$$

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References:

[**P**] P #634, *The College Mathematics Journal,* **(29) #4,** September 1998.